Decays of Hadrons
Selection Rules
Weak decays of light hadrons
CKM matrix
Neutral Meson Mixing
Decays of Hadrons

- The proton is the only completely stable hadron
- The free neutron has a weak decay ($\tau \approx 15$ mins)
- **Decay length** of a particle is the distance it travels before decaying: $L = \beta \gamma c \tau$

<table>
<thead>
<tr>
<th>Force</th>
<th>Typical $\tau$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>QCD</td>
<td>$10^{-20} - 10^{-23}$</td>
</tr>
<tr>
<td>QED</td>
<td>$10^{-20} - 10^{-16}$</td>
</tr>
<tr>
<td>Weak</td>
<td>$10^{-13} - 10^3$</td>
</tr>
</tbody>
</table>

- $\pi^\pm$, $K^\pm$, $K_L^0$ mesons are long-lived ($\tau \approx 10$ ns) and have weak decays
  - Live long enough to travel outside radii of collider detectors ($L \approx 10$ m)
- $K_S^0$ mesons and $\Lambda^0$ hyperons are less long-lived ($\tau \approx 100$ ps) and have weak decays with decay lengths of $L \approx \text{cm}$ which are inside collider detectors

- $\pi^0 \rightarrow \gamma \gamma$, $\eta \rightarrow \gamma \gamma$ are electromagnetic decays, reconstructed from pairs of photons

- $\rho$, $\omega$, $\phi$, $K^*$, $\Lambda$, $\Sigma^*$, $\Xi^*$ are resonances with strong decays.
  - Reconstructed as broad structures with widths $\Gamma \approx 100$ MeV.
## Decay Conservation Laws

- Relevant quantum numbers are:
  - strong isospin \((I, I_3)\)
  - parity \((P)\)
  - quark flavour: described using strangeness \((S=N(s)−N(\bar{s}))\), charm \((C=N(c)−N(\bar{c}))\), beauty \((B=N(b)−N(\bar{b}))\)
- Baryon number and lepton numbers are always conserved!

<table>
<thead>
<tr>
<th></th>
<th>Baryon Number</th>
<th>Strong Isospin, (I)</th>
<th>Strong Isospin, (I_3)</th>
<th>Flavour, (S, C, B)</th>
<th>Parity, (P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>EM</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Weak</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
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</tbody>
</table>
Decays of Charmonium

• The $J/\psi$ meson is a $c\bar{c}$ state. It must decay to particles without charm quarks as $M(J/\psi) < 2M(D)$.

• Two options: decay via three gluons or one photon.

• Strong rate is suppressed by $\alpha_s^6(q_{\text{gluon}})$. This is comparable to $\alpha^2$ for EM decay
  ➔ Both strong and electromagnetic final states have large branching ratios.

• The $J/\psi$ meson lives for a relatively long time, giving rise to narrow resonance in e.g. $e^+e^-\rightarrow \text{hadrons}$.

• Similar phenomena occur in decays of $s\bar{s}$ and $b\bar{b}$ mesons.
Charged Pion Decay

- See problem sheet 1
- \( \pi^+ \) consists of \( u\bar{d} \), lightest charged meson
- Decays via weak force to change quark flavour \( u \rightarrow d \)
  \[ \tau(\pi^+) = 26 \text{ ns} \]
  - CKM matrix element factor \( V_{ud} \).
  - Hadronic decay constant \( f_\pi \sim m_\pi \) to account for finite size of pion

\[
\mathcal{M} = \left[ \bar{u}(d)g_W V_{ud} f_\pi \gamma^\mu (1 - \gamma^5) u(u) \right] \frac{1}{q^2 - m_W^2} \left[ \bar{u}(\nu_\mu)g_W \gamma^\mu (1 - \gamma^5) v(\mu^+) \right]
\]

\[
\approx V_{ud} f_\pi \frac{g_W^2}{m_W^2} \left[ \bar{u}(d)\gamma^\mu (1 - \gamma^5) u(u) \right] \left[ \bar{u}(\nu_\mu)\gamma^\mu (1 - \gamma^5) v(\mu^+) \right]
\]

\[
|\mathcal{M}|^2 = 4 G_F^2 |V_{ud}|^2 f_\pi^2 m_\mu^2 [p_\mu \cdot p_\nu]
\]

\[
\Gamma = \frac{|\vec{p}^*|}{8\pi m_1^2} |\mathcal{M}|^2 = \frac{G_F^2}{8\pi} |V_{ud}|^2 f_\pi^2 m_\mu^2 (m_\pi^2 - m_\mu^2)
\]

\( \mu \) preferred to \( e \)
Charged Kaon Decays

- Charged kaon is $\bar{s} u$ with $m_K = 498$ MeV
- Lightest mesons containing strange quarks $\Rightarrow$ must decay by weak force
  \[ \tau(K^\pm) = 12 \text{ ns} \]
- Leptonic decays
  - $\text{BR}(K^+ \rightarrow \mu^+\nu_\mu) = 63\%$
  - Kaon decay constant, $f_K = 160$ MeV
  - $V_{us} = 0.22$ (Cabibbo angle)
- Semileptonic decays
  - $\text{BR}(K^+ \rightarrow \pi^0\mu^+\nu_\mu) = 3.8\%$
  - $\text{BR}(K^+ \rightarrow \pi^0e^+\nu_e) = 5.1\%$
- Hadronic Decays
  - $\text{BR}(K^+ \rightarrow \pi^0\pi^+) = 21\%$
  - $\text{BR}(K^+ \rightarrow \pi^+\pi^+\pi^-) = 5.6\%$
• **Mass eigenstates** and **weak eigenstates** of quarks are not identical.
  ➡ Decay properties measure mass eigenstates with a definite lifetime and decay width
  ➡ The weak force acts on the weak eigenstates.

• Weak eigenstates are admixture of mass eigenstates, conventionally described using CKM matrix to mix the down-type quarks:

\[
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix} =
\begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}
\]

• *e.g.* weak eigenstate of the strange quark is a mixture between down, strange and bottom mass eigenstates

\[
s' = V_{cd}d + V_{cs}s + V_{cb}b
\]

• The CKM matrix is unitary, \( V_{CKM}^\dagger V_{CKM} = 1 \); standard parameterisation in terms of three mixing angles \( (\theta_1, \theta_2, \theta_3) \) and one complex phase \( (\delta) \) is:

\[
\begin{pmatrix}
  \cos \theta_1 & \sin \theta_1 \cos \theta_3 & \sin \theta_1 \sin \theta_3 \\
  -\sin \theta_1 \cos \theta_3 & \cos \theta_1 \cos \theta_2 \cos \theta_3 - \sin \theta_2 \sin \theta_3 e^{i\delta} & \cos \theta_1 \cos \theta_2 \sin \theta_3 + \sin \theta_2 \cos \theta_3 e^{i\delta} \\
  \sin \theta_1 \sin \theta_2 & -\cos \theta_1 \sin \theta_2 \cos \theta_3 - \cos \theta_2 \sin \theta_3 e^{i\delta} & -\cos \theta_1 \sin \theta_2 \sin \theta_3 + \cos \theta_2 \cos \theta_3 e^{i\delta}
\end{pmatrix}
\]
Nobel Prize in Physics 2008

• Awarded to Makoto Kobayashi, High Energy Accelerator Research Organization (KEK), Tsukuba, Japan and Toshihide Maskawa, Yukawa Institute for Theoretical Physics (YITP), Kyoto University, and Kyoto Sangyo University, Japan

• "for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature"
Experimental Measurements of CKM Matrix

- Many measurements made by the BaBar and Belle experiments.
- Both study $e^+e^− \rightarrow \gamma^{(4s)} \rightarrow B^0 \bar{B}^0$ to measure the decays of $b$ and $c$ quarks, e.g. $V_{cb}$ and $V_{ub}$

<table>
<thead>
<tr>
<th>Exclusive $B \rightarrow \pi \ell \nu$ (as a function of $q^2$)</th>
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$B \rightarrow D^* \ell \nu$ decays (as function of $D^*$ recoil) measures $|V_{cb}| = 0.0374 \pm 0.0017$
The Wolfenstein Parameterisation

• An expansion of the CKM matrix in powers of $\lambda = V_{us} = 0.22$

\[
V_{\text{CKM}} = \begin{pmatrix}
1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \lambda^2/2 & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix} + \mathcal{O}(\lambda^4)
\]

• Parameterisation reflects almost diagonal nature of CKM matrix:
  ➡ The diagonal elements $V_{ud}, V_{cs}, V_{tb}$ are close to 1
  ➡ Elements $V_{us}, V_{cd} \sim \lambda$ are equal and measure $\lambda$
  ➡ Elements $V_{cb}, V_{ts} \sim \lambda^2$ are equal and measure $A$
  ➡ Elements $V_{ub}, V_{td} \sim \lambda^3$ are very small

• Note that the parameter $\rho$ and the complex phase $\eta$ only appear in the very small elements $V_{ub}$ and $V_{td}$, and are thus hard to measure.
Flavour Changing Neutral Currents

• At 1st order, there are no allowed transitions between quarks of the same charge, e.g. $s \leftrightarrow d$, $c \leftrightarrow u$, $b \leftrightarrow s$, $b \leftrightarrow d$

• Weak neutral current (the $Z$ boson) does not change the flavour of fermions.

• At 2nd order so-called “Penguin Diagrams” can cause transitions such as $b \leftrightarrow s$
  • e.g. $b \rightarrow s \bar{s}$, $B^0 \rightarrow \phi K^0$

\[ B^0 \rightarrow J/\psi K^0 \]
Neutral Meson Mixing

- Second order weak interactions mix long-lived neutral mesons with their antiparticles:
  \[ K^0 (\bar{s}d), D^0 (\bar{c}u), B^0 (\bar{b}d), B_s (\bar{b}s) \]

\[ K^0 \leftrightarrow \bar{K}^0 \quad D^0 \leftrightarrow \bar{D}^0 \quad B^0 \leftrightarrow \bar{B}^0 \quad B_s \leftrightarrow \bar{B}_s \]

Observed particles (weak decay eigenstates) are mixtures of flavour eigenstates:

\[ K_S = 1/\sqrt{2} (K^0 + \bar{K}^0) \quad \text{with} \quad \tau_S = 0.09 \text{ ns} \]
\[ K_L = 1/\sqrt{2} (K^0 - \bar{K}^0) \quad \text{with} \quad \tau_L = 51 \text{ ns} \]

Mass difference \( \Delta m_K = m_L - m_S = 3.52(1) \times 10^{-12} \text{ MeV} = 0.53 \times 10^{-10} \text{ s}^{-1} \)

This is the oscillation frequency of the mixing

More about this next week, when we talk about CP violation
• Many measurements, including results from BaBar, Belle, Tevatron and LHCb experiments. Semileptonic $b \rightarrow u$ decays, penguin diagrams, neutral meson mixing and CP violation are used to find best values for $\eta$ and $\rho$ parameters in Wolfenstein parameterisation.
Summary: Decays of Hadrons

- **Strong decays** are characterised by very short lifetimes, $\tau \sim 10^{-20} - 10^{-23}$ s appearing as resonances with a large width $\Gamma \sim \text{MeV}$.
  - Final states are hadronic. All quantum numbers are conserved.
- **Electromagnetic decays** are characterised by $\tau \sim 10^{-20} - 10^{-16}$ s.
  - Decays containing photons are electromagnetic.
  - All quantum numbers conserved except total isospin, $I$.
- **Weak decays** characterised by long lifetimes, $\tau \sim 10^{-13} - 10^3$ s.
  - Only decays that allow change of quark flavour (including s, c, b decays).
  - Responsible for most light meson and baryon decays.
  - Particles can live long enough to reach the detector.
  - Final states may be leptonic, semi-leptonic or hadronic.
  - Strong Isospin, $I, I_3$, Parity, $P$, Flavour quantum numbers not conserved.
- **CKM matrix** relates the quark mass eigenstates to the weak eigenstates
  - Allows higher order penguin diagrams, and neutral meson mixing.
  - Contains four free parameters, including a complex phase (leads to CP violation).