# Recent results in kaon physics 

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Many new results from all the kaon experiments. A choice of topics:

- Vus and CKM unitary test
- RK and LFV tests
- pion-pion scattering lengths and ke4
- and $K^{ \pm} \rightarrow \pi^{ \pm} v v$ proposal


# Semileptonic decays and $V_{u s}$ 

$$
\begin{aligned}
& \text { Semileptonic decays } \\
& \Gamma\left(K_{l 3(\gamma)}\right)=\frac{C_{K}^{2} G_{F}^{2} M_{K}^{5}}{192 \pi^{3}} S_{E W}\left|V_{u s}\right|^{2}\left|f_{+}^{K^{\circ} \pi^{-}}(0)\right|^{2} I_{K l}(\lambda)\left(1+2 \Delta_{K}^{S U(2)}+2 \Delta_{K l}^{E M}\right) \\
& \text { with } K=K^{+}, K^{0} ; l=e, \mu \quad \text { and } \quad C_{K}^{2}=1 / 2 \text { for } K^{+}, 1 \text { for } K^{0} \\
& S_{E W} \quad \text { Universal short distance } \\
& \text { EW correction (1.0232) } \\
& f_{+}{ }^{\circ}{ }^{\circ} \pi(\mathbf{0}) \text { Hadronic matrix element } \\
& \text { at zero momentum transfer ( } t=0 \text { ) } \\
& \Delta_{K}{ }^{S U(\mathbf{2})} \quad \begin{array}{l}
\text { Form factor correction for strong } \\
\\
\mathrm{SU}(2) \text { breaking }
\end{array} \\
& \text { SU(2) breaking } \\
& \Delta_{K l}{ }^{E M} \quad \text { Long distance EM effects } \\
& \Gamma\left(K_{l 3(\gamma)}\right) \begin{array}{c}
\text { Branching ratios with } \\
\text { well determined }
\end{array} \\
& \text { well determined } \\
& \text { treatment of radiative } \\
& \text { decays; lifetimes } \\
& \text { Phase space integral: } \lambda s \\
& \text { parameterize form factor } \\
& \text { dependence on } t \text { : } \\
& K_{e 3} \text { : only } \lambda_{+} \text {(or } \lambda_{+}{ }^{\prime} \lambda_{+}{ }^{\prime \prime} \text { ) } \\
& K_{\mu 3}: \text { need } \lambda_{+} \text {and } \lambda_{0}
\end{aligned}
$$

NA48: $K^{ \pm} \rightarrow \pi^{0{ }^{ \pm}} \boldsymbol{v}$
Arrows indicate signal region


| Total Number of events $\mathrm{K}+/(\mathrm{K}-)$ |  |  |
| :---: | :---: | :---: |
| $\mathrm{Ke} 3:$ | $56 \mathrm{k}(31 \mathrm{k})$ |  |
| $\mathrm{K} \mu 3:$ | $49 \mathrm{k}(28 \mathrm{k})$ |  |
| $\pi^{ \pm} \pi^{0}:$ | $462 \mathrm{k}(257 \mathrm{k})$ |  |


| Ke3: | $6.98 \pm 0.01$ | $(6.94 \pm 0.01) \%$ |
| :--- | :--- | :--- |
| K $\mu 3:$ | $9.27 \pm 0.01$ | $(9.25 \pm 0.01) \%$ |
| PipiO: | $14.18 \pm 0.01$ | $(14.12 \pm 0.01) \%$ |

Background < 1 \%
Trigger efficiency > 99.8 \%

## NA48: Ratios of branching fractions

$R(K e 3 / K 2 \pi)=0.2470 \pm 0.0009($ stat $) \pm 0.0004$ (sys)
$R(K \mu 3 / K 2 \pi)=0.1636 \pm 0.0006($ stat $) \pm 0.0003$ (sys)


Detector acceptance with radiative effects
Particle ID efficiency
Trigger efficiency
Form Factors
Assuming $\operatorname{Br}(K 2 \pi)$ from $P D G$ :

$\operatorname{Br}(K \mu 3)=0.03425 \pm 0.00013($ stat $) \pm 0.00006($ sys $) \pm 0.00020$ (norm) ${ }^{0.16}$
$\operatorname{Br}(\mathrm{Ke} 3)=0.05168 \pm 0.00019($ stat $) \pm 0.00008($ sys $) \pm 0.00030$ (norm)


Compatible with BNL-E865

## NA48: Vus

## Given $\mathrm{Br}(\mathrm{Ke} 3)$ and $\mathrm{Br}(\mathrm{Kmu} 3)$

## $\frac{B R\left(K_{\ell 3}\right)}{\tau_{K}}=\frac{C_{K}^{2} G_{F}^{2} m_{K}^{5}}{192 \pi^{3}} S_{E W}\left|V_{u s}\right|^{2}\left|f_{+}(0)\right|^{2} I_{K}^{\ell}\left(\lambda_{+0}\right)\left(1+\delta_{S U(2)}^{\ell}+\delta_{E M}^{\ell}\right)$

## External Input Used

$M_{K+}$ and $\tau_{K^{+}}$from PDG
$G_{F}=\quad(1.16637 \pm 0.00001)$ $\times 10^{-5} \mathrm{GeV}^{-2}$
$S_{E W}=\quad(1.0230 \pm 0.0003)$
$\delta_{S U 2}^{e, \mu}=(2.31 \pm 0.22) \%$
$\delta_{E M}^{e}=(0.03 \pm 0.10) \%$
$\delta_{E M}^{\mu}=(0.20 \pm 0.20) \%$
$I_{K}^{e}=0.1591 \pm 0.0012$
$I_{K}^{\mu}=0.1066 \pm 0.0008$
$\left|V_{u s}\right| f_{+}(0)=0.21928 \pm 0.00039($ stat $) \pm 0.00017($ sys $) \pm 0.00063$ (norm) $\pm 0.00096$ (ext) Ke 3
$\left|V_{u s}\right| f_{+}(0)=0.21774 \pm 0.00041($ stat $) \pm 0.00019$ (sys) $\pm 0.00064$ (norm) $\pm 0.00103$ (ext) K K 3


$$
\begin{aligned}
& \left|V_{u s}\right| f_{+}(0)=0.2193 \pm 0.0012 \mathrm{Ke3} \\
& \left|V_{u s}\right| f_{+}(0)=0.2177 \pm 0.0013 \mathrm{Ku} 3 \\
& \left|V_{u s}\right| f_{+}(0)=0.2188 \pm 0.0012 \mathrm{Kl3} \\
& \left|V_{u s}\right|_{\text {unitarity }} f_{+}(0)=0.2185 \pm 0.0022 \\
\left|V_{u d}\right|= & 0.9738 \pm 0.0003 \quad\left|V_{u b}\right|=(3.60 \pm 0.7) \times 10^{-3} \\
f_{+}(0)= & 0.961(8)
\end{aligned}
$$

In good agreement with CKM unitarity

## KLOE



## FlaviA $\quad \mid V_{\mathrm{us}} f_{+}(0)$ from $K_{l 3}$ data

Approx. contrib. to \% err from:

|  | - | $K_{L} e 3$ | $0.21638(55)$ | 0.25 | 0.09 | 0.19 | 0.10 | 0.10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $K_{L} \mu 3$ | $0.21678(67)$ | 0.31 | 0.10 | 0.18 | 0.15 | 0.15 |
|  |  | $K_{s} e 3$ | $0.21554(142)$ | 0.66 | 0.65 | 0.03 | 0.10 | 0.10 |
|  |  | $K^{ \pm} e 3$ | 0.21746(85) | 0.39 | 0.29 | 0.09 | 0.24 | 0.09 |
|  |  | $K^{ \pm} \mu 3$ | $0.21810(114)$ | 0.52 | 0.42 | 0.09 | 0.26 | 0.15 |
| $\begin{array}{lllll}0.214 & 0.216 & 0.218 & 0.22\end{array}$ |  |  |  |  |  |  |  |  |
| Average: $\left\|V_{u s}\right\| f_{+}(0)=0.21668(45)$ |  |  |  | $\chi^{2 / n d f}=2.74 / 4(60 \%)$ |  |  |  |  |
| $\Delta^{S U(2)}{ }_{\exp }=2.86(38) \%$ |  |  | $\rightarrow \underset{\substack{\text { success of CHPT } \\ \text { calculations }}}{\left[\Delta^{S U(2)}{ }_{\text {th }}=2.31(22) \%\right] ~}$ |  |  |  |  |  |

## $K_{l 3}$ average: $\left|V_{u s}\right| f_{+}(0)=\mathbf{0 . 2 1 6 6 8 ( 4 5 )}$

$-0.1 \%$ respect to CKM06 and PDG06
Leutwyler \& Roos '84 Conventional choice for value of $f_{+}(0)$ until a $f_{+}(0)=0.961(8) \quad$ definitive evaluation becomes available

$$
K_{l 3} \text { average: }\left|V_{u s}\right|=0.2255(19)
$$

Marciano \& Sirlin '06

$$
\left|V_{u d}\right|=0.97377(27)
$$

Average from $0^{+} \rightarrow 0^{+} \beta$ decays with recent evaluation of EW radiative corrections

$$
V_{u d}{ }^{2}+V_{u s}{ }^{2}-1=-0.0009(10)
$$

Compatibility with unitarity $-0.9 \sigma$

For each state of kaon charge, we evaluate:

$$
r_{\mu e}=\frac{\left(R_{\mu e}\right)_{\mathrm{obs}}}{\left(R_{\mu e}\right)_{\mathrm{SM}}}=\frac{\Gamma_{\mu 3}}{\Gamma_{e 3}} \cdot \frac{I_{e 3}\left(1+\delta_{e 3}\right)}{I_{\mu 3}\left(1+\delta_{\mu 3}\right)}=\frac{\left[\left|V_{u s}\right| f_{+}(0)\right]_{\mu 3, \mathrm{obs}}^{2}}{\left[\left|V_{u s}\right| f_{+}(0)\right]_{e 3, \mathrm{obs}}^{2}}=\frac{\left(G_{F}^{\mu}\right)^{2}}{\left(G_{F}^{e}\right)^{2}}
$$

## $K^{ \pm}$modes <br> $r_{\mu e}=1.0059(87)$

Using 2004 BRs*
$r_{\mu e}=1.019(13)$

$$
\begin{gathered}
K_{L, S} \text { modes } \\
r_{\mu e}=1.0039(56)
\end{gathered}
$$

Using 2004 BRs*

$$
r_{\mu e}=1.054(15)
$$

Compare sensitivity from $\pi \rightarrow l v$ decays:

$$
\left(r_{e \mu}\right)_{\pi / 2}=0.9966(30)
$$

see Erler, Ramsey-Musolf '06
*Assuming current values for form-factor slopes and $\Delta^{\mathrm{EM}}$

$$
\text { Compare also to : } \quad\left(\left(g_{\mu}^{2} / g_{e}^{2}\right)_{\tau \rightarrow l \nu \bar{\nu}}=0.9998(40)\right)
$$

$f_{+}(0)$ from LR 84
$\left|V_{u s}\right|=0.2255(19)$ from $K l 3$


Fit results, no constraint:

$$
\begin{gathered}
V_{u d}=0.97377(27) \\
V_{u s}=0.2245(16) \\
\chi^{2} / \mathrm{ndf}=0.75 / 1(39 \%)
\end{gathered}
$$

Fit results, unitarity constraint:

$$
\begin{gathered}
V_{u d}=0.97403(22) \\
V_{u s}=0.2264(9) \\
\chi^{2} / \mathrm{ndf}=3.13 / 2(\mathbf{2 1 \%})
\end{gathered}
$$

Agreement with unitarity at $1.3 \sigma$

Form Factors

KI3 matrix element:
$\mathcal{M} \propto \mathbf{f}_{+}\left(\mathbf{q}^{2}\right)\left(\mathbf{p}_{\mathbf{K}}+\mathbf{p}_{\pi}\right)^{\mu} \overline{\mathbf{u}}_{\mathbf{l}} \gamma_{\mu}\left(1+\gamma_{5}\right) \mathbf{u}_{\mathbf{v}}+\mathbf{f}_{-}\left(\mathbf{q}^{2}\right) \mathbf{m}_{1} \overline{\mathbf{u}}_{1} \gamma_{\mu}\left(1+\gamma_{5}\right) \mathbf{u}_{\mathbf{v}}$

Scalar form factor:

$$
\mathrm{f}_{0}\left(\mathrm{q}^{2}\right)=\mathrm{f}_{+}\left(\mathrm{q}^{2}\right)+\frac{\mathrm{q}^{2}}{\mathrm{~m}_{\mathrm{K}}^{2}-\mathrm{m}_{\pi}^{2}} \mathrm{f}_{-}\left(\mathrm{q}^{2}\right)
$$

Linear/Quadratic expansion:

$$
\begin{aligned}
f_{+}\left(q^{2}\right) & =f_{+}(0)\left(1+\lambda_{+}^{\prime} \frac{q^{2}}{m_{\pi+}^{2}}+\frac{1}{2} \lambda_{+}^{\prime \prime} \frac{q^{4}}{m_{\pi+}^{4}}\right) \\
f_{0}\left(q^{2}\right) & =f_{+}(0)\left(1+\lambda_{0} \frac{q^{2}}{m_{\pi^{+}}^{2}}\right)
\end{aligned}
$$

Slopes from Ke3

## KTeV KLOE ISTRA+ NA48 FlaviaNet fit



Slope parameters $\times 10^{3}$

$$
\lambda_{+}^{\prime}=25.15 \pm 0.87
$$

$$
\lambda_{+}^{\prime \prime}=1.57 \pm 0.38
$$

$$
\rho\left(\lambda_{+}^{\prime}, \lambda^{\prime \prime}\right)=-0.941
$$

$$
\chi^{2} / \mathrm{ndf}=5.3 / 6 \quad(51 \%)
$$

Excellent compatibility Significance of $\lambda^{\prime \prime}{ }_{+} \sim 4 \sigma$

## KTeV

Each ellipse is from the average of Ke 3 and $\mathrm{K} \mu 3$

## - lavi $\boldsymbol{A}$




$\lambda_{+}^{\prime}=(24.8 \pm 1.1) \times 10^{-3} \lambda^{\prime \prime}{ }_{+}=(1.64 \pm 0.44) \times 10^{-3} \lambda_{0}=(13.4 \pm 1.2) \times 10^{-3}$ $P\left(\chi^{2}\right) \sim 10^{-6}$
$R_{K}$ and LFV

## $\mathrm{R}_{\mathrm{K}}=\Gamma\left(\mathrm{K}^{+} \rightarrow \mathrm{e}^{+} \mathrm{v}\right) / \Gamma\left(\mathrm{K}^{+} \rightarrow \mu^{+} v\right)$

$$
R_{M}:=\frac{\Gamma\left(M \rightarrow e \nu_{e}(\gamma)\right)}{\Gamma\left(M \rightarrow \mu \nu_{\mu}(\gamma)\right)}=\frac{m_{e}^{2}}{m_{\mu}^{2}}\left(\frac{m_{M}^{2}-m_{e}^{2}}{m_{M}^{2}-m_{\mu}^{2}}\right)^{2}\left(1+\delta R_{M}\right)
$$

| PDG value | $2.45 \pm 0.11$ |
| :--- | :---: |
| SM prediction | $2.472 \pm 0.001$ |
| NA48/2 (2003) | $2.416 \pm 0.043_{\text {(stat) }} \pm 0.024_{\text {(syst) }}$ |$\quad$| $\delta R_{M}$ from radiative corrections |
| :---: |$\quad$ For K ${ }^{ \pm}: \delta R_{K}=-(3.78 \pm 0.04) \%$

$$
R_{K}^{L F V}=\frac{\sum_{i} K \rightarrow e \nu_{i}}{\sum_{i} K \rightarrow \mu \nu_{i}} \simeq \frac{\Gamma S M\left(K \rightarrow e \nu_{e}\right)+\Gamma\left(K \rightarrow e \nu_{\tau}\right)}{\Gamma_{S M}\left(K \rightarrow \mu \nu_{\mu}\right)}, \quad i=e, \mu, \tau
$$

| Masiero, |
| :--- |
| Paradisi, |
| Petronzio, |
| hep-ph/0511289 |

$L F V \tau$ decay of $O\left(10^{-10}\right)$

$\Delta_{R}^{31} \sim \frac{\alpha_{2}}{4 \pi} \delta_{R R}^{31} \quad \begin{aligned} & \text { out-of-diag slepton } \\ & \text { mixing matrix }\end{aligned}$

$$
\Delta_{R}^{31} \sim 5 \cdot 10^{-4} t_{\beta}=40 M_{H^{ \pm}}=500 \mathrm{GeV}
$$

$\Delta_{K}^{e-\mu}{ }_{K S Y} \simeq\left(\frac{m_{K}^{4}}{M_{H^{ \pm}}^{4}}\right)\left(\frac{m_{\tau}^{2}}{m_{e}^{2}}\right)\left|\Delta_{R}^{31}\right|^{2} \tan ^{6} \beta \approx 10^{-2}$

Missing mass vs momentum (MC)


- The dominant background is $\mathrm{K} \mu 2$
- Measured from the data in momentum bins

The theory prediction for $R_{K}$ includes the IB term from $\mathrm{KI} 2 \gamma$ decays
Radiative corrections applied according to the prescription of M . Finkemeier:
(Phys.Lett.B387:391-394,1996) using the matrix elements from J. Bijnens et al (Nucl.Phys. B396 (1993) 81-118)
proper treatment of radiative correction is important

## Total Ke2 events: $\left(3407\right.$ を $63_{\text {stat }}$ を $54_{\text {syst }}$ )

$$
R_{K}=\frac{N_{\text {Ke2raw }}-N_{\text {Ke2back }}}{\operatorname{TrEff}(\mathrm{Ke} 2) * \operatorname{Acc}(\mathrm{Ke} 2) * C_{e}} * \frac{\operatorname{Acc}(K \mu 2) * C_{\mu}}{D *\left(N_{K \mu 2 \mathrm{raw}}-N_{K \mu 2 \mathrm{back}}\right)}
$$

| $\mathbf{N}_{\text {kl2raw }}$ | Raw KI2 events |
| :--- | :--- |
| $\mathbf{N}_{\text {kl2back }}$ | Background in KI2 |
| TrEff(Ke2) | Ke2 trigger efficiency |
| $\mathrm{C}_{1}$ | Losses due to E/p cut |
| Acc(KI2) | KI2 acceptance |

NB: The dominant contribution to the systematics, the background subtraction error, scales with the statistics

| Standard Model | $(2.472 \pm 0.001)^{*} 10^{-5}$ |
| :--- | :--- |
| PDG | $(2.45 \pm 0.11) * 10^{-5}$ |
| NA48: 2004 data | $(2.455 \pm \mathbf{0 . 0 4 5} \pm \mathbf{0 . 0 4 1}) * 10^{-5}$ |



2007 run to reach 0.3\% precision


$$
l \mathbf{H}^{ \pm} \nu_{\tau} \rightarrow \frac{\mathbf{g}_{2}}{\sqrt{2}} \frac{\mathbf{m}_{\tau}}{\mathbf{M}_{\mathrm{W}}} \Delta_{13} \tan ^{2} \beta
$$

$$
\Delta_{3 j} \sim \frac{\alpha_{2}}{4 \pi} \delta_{3 j} \begin{aligned}
& \text { slepton } \\
& \text { flavour mixing } \\
& \text { angle }
\end{aligned}
$$



- Number of $\mathrm{K}_{\mu 2}$ events
- $\mathrm{N}_{\mathrm{k} \mid \mathrm{L} 2}=499251584 \pm 35403$
- Number of $\mathrm{K}_{\mathrm{e} 2}$
- $N_{\text {e2 }}=8090 \pm 156$

| Systematics(fractional): |  |
| :--- | :--- |
| - IB | 0.0005 |
| - IB/DE | 0.003 |
| - TRK+VTX | 0.009 |
| - PID | $0.009 \pm 0.015$ |
| - TRG | $0.006 \pm 0.004$ |

Present statistical accuracy $1.9 \%$
Final statistics will be $\times 1.3$, counting $>10 \mathrm{k}$ events
Present stat error dominated by background:

- Signal fluctuation 1.1\%
- MC statistics (1.4\%) $\oplus$ background fluctuation (0.7\%)
$1 \mathrm{fb}^{-1}$ of additional MC statistics under production
Cuts still have to be tuned, PID can be improved

$$
\begin{aligned}
& R=(2.55 \pm 0.55 \pm 0.55) \times 10^{-5} \\
& S M R=(2.472 \pm 0.001) \times 10^{-5}
\end{aligned}
$$

## Low-energy QCD

## NA48: $\pi \pi$ scattering in $K^{ \pm} \rightarrow \pi^{ \pm} \pi^{0} \pi^{0}$

Observation of a cusp structure in the $\pi^{\circ} \pi^{\circ}$ invariant mass distribution at $M_{\pi^{\circ} \pi^{\circ}}=2 m_{\pi+}$ : an unexpected discovery from the NA48/2


$\mathrm{M}_{00}$ resolution $\sigma=0.56 \mathrm{MeV}$ at $\mathrm{M}_{00}=2 \mathrm{~m}_{+}$

$\pi \pi$ charge exchange amplitude near threshold is proportional to the difference of scattering lengths a0-a2
(Cabibbo PRL 93, 2004)

## Fit results:

$$
\begin{gathered}
\left(a_{0}-a_{2}\right) m_{+}=0.261 \pm 0.006 \pm 0.003 \pm 0.0013 \pm 0.013 \\
\text { (stat.) } \begin{array}{c}
\text { (syst.) } \\
a_{2} m_{+}=-0.037 \pm 0.013 \pm 0.009 \pm 0.002 \text { (ext.) }
\end{array} \text { (theor.) }
\end{gathered}
$$

## External uncertainty:

from the uncertainty on the ratio of $\mathrm{K}^{+} \rightarrow \pi^{+} \pi^{+} \pi^{-}$and $\mathrm{K}^{+} \rightarrow \pi^{+} \pi^{\circ} \pi^{\circ}$ decay widths Theoretical uncertainty on $\left(a_{0}-a_{2}\right) m_{+}: \pm 5 \%$
(estimated effect from neglecting higher order diagrams and radiative corrections)

Fit with analiticity and chiral symmetry constraint between $a_{0}$ and $a_{2}$ (Colangelo, Gasser, Leutwyler, PRL 86 (2001) 5008)

$$
\left(a_{0}-a_{2}\right) m_{+}=\underset{\text { (stat.) }}{0.263 \pm} \underset{\text { (syst.) }}{0.003} \underset{\text { (ext.) }}{0.0014} \pm \underset{\text { (theor.) }}{0.0013} \underset{0.013}{0.00}
$$

Pionium mean lifetime $\tau_{1 s}=\left(2.91_{-0.43}^{+0.24}\right) \times 10^{-15} \mathrm{~s} \quad$ Good agreement
DIRAC $\longleftrightarrow\left|a_{0}-a_{2}\right| m_{+}=0.264_{-0.011}^{+0.020}$

## NA48: $\pi \pi$ scattering in $K^{ \pm} \rightarrow \pi^{+} \pi^{-} e^{ \pm} \nu$

A rare decay [ B.R. $=(4.09 \pm 0.09) \times 10^{-5}$ ] described by five independent variables


Cabibbo - Maksymowicz variables : $\begin{gathered}s_{\pi} \equiv \mathrm{M}_{\pi \pi}{ }^{2}{ }_{2}{ }_{s_{\mathrm{e}}} \equiv \mathrm{M}_{\mathrm{ev}}\end{gathered}$

$$
\begin{array}{ll}
\theta_{\mathrm{e}} & \text { For } \mathrm{K}^{+} \Rightarrow \mathrm{K}^{-} \\
\theta_{\pi} & \phi \Rightarrow \pi+\phi \\
\phi & \theta_{\mathrm{e}} \Rightarrow \pi-\theta_{\mathrm{e}}
\end{array}
$$

$$
\begin{aligned}
& \text { partial wave expansion of the amplitude: expansion in powers of } q^{2}, S e / 4 m \pi^{2} \\
& F, G=\text { Axial Form Factors } \\
& F=F_{s} e^{i \delta s}+F_{p} e^{i \delta p} \cos \theta_{\pi}+d \text {-wave term... } \\
& G=G_{p} e^{i \delta g}+d \text {-wave term... } \\
& \text { H = Vector Form Factor } \\
& H=H_{p} e^{i \delta h}+d \text {-wave term... } \\
& \text { ( } \left.q^{2}=\left(S_{\pi} / 4 m_{\pi}^{2}-1\right)\right) \\
& F_{s}=f_{s}+f_{s}^{\prime} q^{2}+f_{s}^{\prime \prime} q^{4}+f_{e}\left(S_{e} / 4 m_{\pi}^{2}\right)+. \\
& F_{p}=f_{p}+f_{p}^{\prime} q^{2}+. . \\
& G_{p}=g_{p}+g_{p}^{\prime} q^{2}+. . \\
& H_{p}=h_{p}+h_{p}^{\prime} q^{2}+. .
\end{aligned}
$$

Fit parameters: $F_{s} \quad F_{p} \quad G_{p} \quad H_{p}$ and $\delta=\delta_{s}-\delta_{p}$
Ten independent fits, one in each $M_{\pi \pi}$ bin. This allow a model independent analysis Without the overall normalization, one can quote relative form factors and their variation with $q^{2}$
Fs is obtained from relative bin to bin normalization data/MC after fit

To relate scattering lengths to $\delta$, external data and theoretical work needed An example is numerical solution of Roy equations (DFGS EPJ C24, 2002) The centre line parameterization corresponds to a 1-param fit with fixed relation $a_{0}{ }^{2}=f\left(a_{0}{ }^{0}\right)$


Geneva - Saclay: ~ 30,000 events, $p_{K^{+}}=2.8 \mathrm{GeV} / \mathrm{c}$
BNL E865: 406,103 events (with ~ 4.4\% background), $p_{\mathrm{K}^{+}}=6 \mathrm{GeV} / \mathrm{c}$ NA48/2: 677,510 events (with ~0.5\% background), $p_{k^{ \pm}}=60 \mathrm{GeV} / \mathrm{c}$ (the isospin - breaking corrections reduce $\delta$ by $0.01-0.012$ )

One can correct measured Ke4 for isospin symmetry breaking before extracting $a_{0}{ }^{0}$ (Gasser, 2007)


One can correct measured Ke4 for isospin symmetry breaking before extracting $a_{0}{ }^{0}$ (Gasser, 2007)


Ke4:<br>NA48/2 - BNL E865<br>comparison

## Future experiments: $K^{ \pm} \rightarrow \pi^{ \pm} \nu v$

Given the great phenomenological success of the SM up to LEP energies and the limitations/unsatisfactory aspects of the model above the e.w. scale $\Rightarrow$ natural to consider the SM as an effective theory
or the low-energy limit of a more fundamental theory with new degrees of freedom appearing above some energy threshold $\Lambda$

High-energy experiments are the key tool to determine the energy scale of the new d.o.f. via their direct production
Low-energy experiments are a fundamental ingredient to determine the symmetry properties of the new d.o.f. via indirect effects in precisions observables

Precision measurements in the flavour sector allow us to study the flavour symmetries of physics beyond the SM

Rare FCNC decays and $\Delta F=2$ processes are the oservable more sensitive to new flavour-breaking couplings

## $K \rightarrow \pi v v:$ SM Theoretical Prediction

NLO Calculation:


Buchalla \& Buras: 1993, 1999
Misiak, Urban: 1999

$$
\begin{array}{ll}
\lambda=V_{u s} & B\left(K^{+} \rightarrow \pi^{+} v \bar{v}\right)=\kappa_{+} \cdot\left[\left(\frac{\operatorname{Im} \lambda_{t}}{\lambda^{5}} X\left(x_{t}\right)\right)^{2}+\left(\frac{\operatorname{Re} \lambda_{t}}{\lambda^{5}} X\left(x_{t}\right)+\frac{\operatorname{Re} \lambda_{c}}{\lambda} P_{c}(X)\right)^{2}\right] \\
\lambda_{c}=V_{c S}^{*} V_{c d} & \\
\lambda_{t}=V_{t s}^{*} V_{t d} \quad B\left(K_{\mathrm{L}}^{0} \rightarrow \pi^{0} v \bar{v}\right)=\kappa_{\mathrm{L}} \cdot\left(\frac{\operatorname{Im} \lambda_{t}}{\lambda^{5}} X\left(x_{t}\right)\right)^{2} & \text { charm contribution } \\
& \text { contributions }
\end{array} \begin{array}{ll} 
& \text { NNLO } \\
\kappa_{+}=r_{K^{+}} \cdot \frac{3 \alpha^{2} B r\left(K^{+} \rightarrow \pi^{0} e^{+} v\right)}{2 \pi^{2} \sin ^{4} \theta_{W}} \cdot \lambda^{8} & \text { Haisch, Gorbahn, Nierste }
\end{array}
$$




- No SM tree-level contribution
- Strong suppression within the SM because of CKM hierarchy
- Predicted with high precision within the SM at the short-distance level

$$
\underset{\text { hadronic matrix element }}{\mathrm{A}(K \rightarrow \pi \vee \mathrm{~V})=f\left(\mathrm{c}_{\mathrm{SM}} \frac{y_{t}^{2} V_{\mathrm{tS}}^{*} V_{\mathrm{td}}}{16 \pi^{2} \mathrm{M}_{\mathrm{W}}^{2}}+\mathrm{c}_{\mathrm{new}} \frac{\Delta_{\mathrm{sd}}}{\Lambda^{2}} ; \delta_{\text {long }}\right)}
$$

$\underset{2 / 11 / 2007}{\operatorname{from}} B R\left(K^{+} \rightarrow \pi^{0} e^{+} v\right)$
energy scale
of new d.o.f

## Minimal Flavour Violation

flavour symmetry broken only by the (SM) Yukawa couplings

## $\downarrow$

*Small deviations (10-20\%) from SM

- Stringent correlations among the two $K \rightarrow \pi \nu \nu$ modes and a few rare B decays $\left[B \rightarrow K \nu v, B_{\mathrm{s}, \mathrm{d}} \rightarrow l^{+} l^{-}\right]$

A precise exp. info on one of the two $K \rightarrow \pi \nu \nu$ modes is a key ingredient to verify or disproof the MFV hypothesis

## New sources of Flavour Symmetry

breaking around the TeV scale
$\downarrow$
-Potentially large effects, especially in the three CPV $K_{\mathrm{L}}$ decays (no $\lambda^{5}$ suppression)

* Correlations with observables in B physics not obvious

In presence of sizable non-MFV couplings mandatory to explore also the $K_{L} \rightarrow \pi l l$ modes
*Non-standard effects induced by chargino-squarks amplitudes largely dominant in $\mathrm{K} \rightarrow \pi \nu \nu$ with respect to similar effects in B physics
*The A terms are still largely unconstrained squark-sector trilinear terms


SM expectation $=(8.0 \pm 1.1) \times 10^{-11}$ dominated by $C K M$ uncertainty 3 events E787/E949: $B R\left(K^{+} \rightarrow \pi^{+} v v\right)=1.47^{+1.30}{ }_{-0.89} \times 10^{-10}$


## NA62 Detector Layout



## Backeground rejection



## 1) Kinematical Rejection

$$
m_{m i s s}^{2} \approx m_{K}^{2}\left(1-\frac{\left|P_{\pi}\right|}{\left|P_{K}\right|}\right)+m_{\pi}^{2}\left(1-\frac{\left|P_{K}\right|}{\left|P_{\pi}\right|}\right)-\left|P_{K} \| P_{\pi}\right| \vartheta_{\pi K}^{2}
$$

2) Photon vetoes to reject $K^{+} \rightarrow \pi^{+} \pi^{0}:$
$\mathrm{P}\left(\mathrm{K}^{+}\right)=75 \mathrm{GeV} / \mathrm{c}$
Requiring $\mathrm{P}\left(\pi^{+}\right)<35 \mathrm{GeV} / \mathrm{c}$ $P\left(\pi^{0}\right)>40 \mathrm{GeV} / \mathrm{c} \quad$ It can be hardly missed in the calorimeters!!


I: $0<m<0.01$
3) PID (RICH) for $K^{+} \rightarrow \mu^{+} v$ rejection

II: $0.026<m<0.068$

## Non kinematically constrained backgrounds



Veto rejection and particle identification are essential

## Conclusions

Many new results from all the kaon experiments:

- Vus and CKM unitary test : compatibility with unitary at -0.9 $\sigma$
- RK and LFV tests: sensitivity of $2 \%$ reached but more data to come, so far no sign of LFV
- pion-pion scattering lengths from cusp and ke4 agree, limited by theoretical uncertainty

I didn't have time to show results on radiative decays and test of $\chi P T$ and CPV:
$K^{ \pm} \rightarrow \pi^{ \pm} \pi^{0} \gamma$ : first evidence for interference between IB and DE contributions $B R\left(K_{L} \rightarrow \pi^{0} \gamma \gamma\right)$ : KTeV and NA48 agree, also with $\chi$ PT $B R\left(K_{S} \rightarrow \gamma \gamma\right)$ : KLOE and NA48 disagree CPV charge kaon asymmetry reach a sensitivity of $10^{-4}$, and new measurement of $\eta_{+-}$

Future Kaon experiments to measure rare decays
$K^{0} \rightarrow \pi^{0} v v$ (JPARC) and $K^{ \pm} \rightarrow \pi^{ \pm} v v$ (CERN)
Kaon physics is still very much alive!

Spares:

Spares: Radiative decays


## What's new in NA48/2 measurement?

> Simultaneous $\mathrm{K}^{+}$and $\mathrm{K}^{-}$beams -> check for CP-Violation
> Enlarged $T^{*}{ }_{\pi}$ region in the low energy part ( $0 \div 80 \mathrm{MeV}$ )
> Negligible background contribution ( $<1 \%$ of the DE component)
> $\gamma$ miss-tagging probability ~ \%。
 for IB, DE and INT

After all cuts the background estimation is $<1 \%$ of DE and can be explained in terms of $K^{ \pm}->\pi^{ \pm} \pi^{0} \pi^{0}$

Use extended Maximum Likelihood for $0.2<W<0.9$ to fit in the region $0 \mathrm{MeV}<T_{\pi}^{\star}<80 \mathrm{MeV}$ (based on $124 \cdot 10^{3}$ events)
Fit performed with free INT term Systematics dominated by trigger efficiency
-> First evidence of Interference between Inner Bremsstrahlung and Direct Emission amplitudes

$\operatorname{Frac}(D E)=\left(3.35 \pm 0.35_{\text {stat }} \pm 0.25_{\text {syst }}\right) \%$
$\operatorname{Frac}($ INT $)=\left(-2.67 \pm 0.81_{\text {stat }} \pm 0.73_{\text {syst }}\right) \%$

Preliminary

## KLOE: $K_{S} \rightarrow \gamma \gamma$

- It is a good test for ChPT (PRD 49 (1994) 2346)
- Experimental value of the BR changed along the years
- From 2003 it is known with a small error (3\%) :
$\operatorname{BR}\left(K_{S} \rightarrow \gamma \gamma\right)=(2.71 \pm 0.06 \pm 0.04) \times 10^{-6}$ due to a measurement of NA48/1 collaboration
- Differs from ChPT O(p ${ }^{4}$ ) by $30 \%$ (possible large $\mathrm{O}\left(\mathrm{p}^{6}\right)$ contribution).


In NA48, the $\mathrm{K}_{\mathrm{L}} \rightarrow \gamma \gamma$ background is a relevant component of the fit.
In KLOE , the background from $\mathrm{K}_{\mathrm{L}}$ is reduced to 0 (tagging).
First measurement of this decay with a pure $K_{S}$ beam.

From 1.6 fb $^{-1}$

To extract the number of signal, the 2D-plot in data is fit using signal and background shapes from MC
$\mathbf{N}_{\text {sig }}=600.3 \pm 34.8$
(5.8\% stat. error)
-• DATA
-- MC all
$\square$ Signal
Background



Background dominated by $K_{s} \rightarrow 2 \pi^{0}$

$$
B R\left(K_{S} \rightarrow \gamma \gamma\right)=N_{\gamma \gamma} \times \frac{\varepsilon_{2 \pi^{0}}\left(\text { tot } \mid K_{L}-\text { crash }\right)}{\varepsilon_{S I G}\left(\text { tot } \mid K_{L}-\text { crash }\right)} \times \frac{B R\left(K_{S} \rightarrow 2 \pi^{0}\right)}{N_{2 \pi^{0}}}
$$

- Where for the signal:

$$
\begin{aligned}
\varepsilon_{\text {SIG }}\left(\text { tot } \mid K_{L}-\text { crash }\right) & =\varepsilon(\text { presel }) \times \varepsilon(\text { veto }) \times \varepsilon\left(\chi^{2}\right)= \\
& =(50.8 \pm 0.6) \%
\end{aligned}
$$

- For the normalization sample, we count events with 4 prompt photons:

$$
\begin{aligned}
& \varepsilon_{2 \pi 0}\left(\text { tot } \mid K_{L}-\text { crash }\right)=\left(65.0 \pm 0.2_{\text {stat }} \pm 0.1_{\text {sys }}\right) \% \\
& \mathbf{N}_{2 \pi 0}=159.8 \text { Mevts }
\end{aligned}
$$

Systematics mainly due to application of data-MC correction curve for cluster efficiency. Cross checked with counting (3-5) prompt photons (159.5 Mevts)

$$
B R\left(K_{S} \rightarrow \gamma \gamma\right)=\left(2.27 \pm 0.13_{\text {stat }}+0.0 .04\right) \times 10^{-6}
$$



## KLOE: $K e 3 \gamma$

We measure $\mathrm{R}=\mathbf{B R}\left(\operatorname{Ke} 3 \gamma ; \mathrm{E}_{\gamma}^{*}>30 \mathrm{MeV}, \boldsymbol{\theta}_{\text {lep } \gamma}>20^{\circ}\right) / \operatorname{BR}(\operatorname{Ke} 3(\gamma))$, using a $328 \mathrm{pb}^{-1}$ 2001-2002 data sample ; Both IB and DE emission contribute to R;
Separation between IB and DE never measured ${ }^{(*)}$; for the first time the DE contribution is measured ;
What needs: $E_{\gamma}^{*}-\theta^{*}$ ele- analysis + low BKG


## Monte Carlo Reliability

- $B R\left(\mathrm{~K}_{\mathrm{e} 3 \mathrm{r}}\right)$ is largely dominated by the IB , as the DE contribution via IB-DE interference is $\sim 1 \%$ level (pure DE is negligibly). DE e ects becomes more significant at high energy, but the number of events is severely reduced.
- KLOE MC ${ }^{(1)}, O\left(p^{2}\right)$ accuracy ~ few \% for $\mathrm{K}_{\text {e3r }}$ after integration, but DE contribution $\sim 1 \% \mathrm{IB}$ -> $\delta(\mathrm{DE}) \sim 100 \%$
- We use a stand alone MC production for IB and $D E, O\left(p^{6}\right)^{(2)}$
(1) C.Gatti, "Monte Carlo Simulation for radiative kaon decay" Eur.Phys. J C45 (2006) 417
${ }^{(2)}$ J. Gasser, B. Kubis, N. Paver, M. Verbeni Eur.Phys. J C40 (2005) 205


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Inputs => 4 MC shapes
free parameters $=\mathbf{I B}+\mathbf{B 1}+\mathbf{D E}$ normalization
fixed $=B 2$, from MC normalized to Data
Goodness of fit $\Rightarrow \chi^{2} /$ dof $=60 / 69$

## $R=\left(924 \pm 23_{\text {stat }} \pm 16_{\text {syst }}\right) \times 10^{-5}$

- DE: first measurement of DE contribution; it is in agreement with $\chi$ PT@O(p) prediction ;
- R : our accuracy on R is not sufficient to solve experimental disagreement;

Gasser J. et ol, Eur.Phys. J C40 (2005) 205


- KTeV measurement refers to a phenomenological model for DE , the FFS model ${ }^{(1)}$, based on four parameters. No enough sensitivity to measure all parameters -> soft kaon approximation ;


## $\mathrm{KTeV}: K_{L} \rightarrow \pi^{0} \gamma \gamma$



- $\mathcal{O}\left(p^{4}\right)$ chiral perturbation calculations
- No free parameters $\rightarrow \mathrm{BR}\left(K_{L} \rightarrow \pi^{0} \gamma \gamma\right)=0.6 \times 10^{-6}$
- Prediction low by factor of 2-3.
- $\mathcal{O}\left(p^{6}\right)$ calculations increase rate.
- Addition of VMD terms further increases rate ( $a_{V}$ ).

IK IK $^{2}$

- Major background comes from $3 \pi^{0}$ decays with 4 clusters in the calorimeter.
- With missing $\gamma$, event reconstructs downstream.
- Photon vetoes help reduce this background.

Ful/ data set



Candidates: 1982, Background: 601, $K_{L} \rightarrow 2 \pi^{0}$ events: 919,322

- Underestimate of background led to higher value in previous KTeV result.
- New results consistent with published NA48 result.
- Result supercedes previous KTeV result.
- All BR adjusted to new $K_{L} \rightarrow \pi^{0} \pi^{0}$ BR.



2/11/2007

## Spares: Cusp

## NA48/2 (PRELIMINARY)

$59,624,170$ fully reconstructed $K^{ \pm} \rightarrow \pi^{ \pm} \pi^{\circ} \pi^{\circ}$ events $\times 10^{2}$


Fit results:

$$
\begin{aligned}
& \qquad \begin{array}{c}
\left(a_{0}-a_{2}\right) m_{+}=0.261 \pm 0.006 \pm 0.003 \pm 0.0013 \pm 0.013 \\
\text { (stat.) } \\
a_{2} m_{+}=-0.037 \pm 0.013 \pm 0.009 \pm 0.002
\end{array} \\
& \text { (syst.) }
\end{aligned}
$$

## External uncertainty:

from the uncertainty on the ratio of $\mathrm{K}^{+} \rightarrow \pi^{+} \pi^{+} \pi^{-}$and $\mathrm{K}^{+} \rightarrow \pi^{+} \pi^{\circ} \pi^{\circ}$ decay widths:

$$
\begin{equation*}
\frac{\Gamma\left(K^{+} \rightarrow \pi^{+} \pi^{+} \pi^{-}\right)}{\Gamma\left(K^{+} \rightarrow \pi^{+} \pi^{0} \pi^{0}\right)}=3.182 \pm 0.047 \tag{PDG2006}
\end{equation*}
$$

giving $\frac{A\left(K^{+} \rightarrow \pi^{+} \pi^{+} \pi^{-}\right)}{A\left(K^{+} \rightarrow \pi^{+} \pi^{0} \pi^{0}\right)}=1.975 \pm 0.015$ at the Dalitz plot centres $(u=v=0)$
(exact isospin symmetry predicts 2)
Theoretical uncertainty on $\left(a_{0}-a_{2}\right) m_{+}: \pm 5 \%$
(estimated effect from neglecting higher order diagrams and radiative corrections)
Fit with analiticity and chiral symmetry constraint between $a_{0}$ and $a_{2}$ (Colangelo, Gasser, Leutwyler, PRL 86 (2001) 5008)

$$
\left(a_{0}-a_{2}\right) m_{+}=\underset{\text { (stat.) }}{0.263 \pm} 0 \underset{\text { (syst.) }}{0.003} \underset{\text { (ext.) }}{0.0014} \pm \underset{\text { (theor.) }}{0.0013} 0 \pm 013
$$

Pionium mean lifetime $\quad \tau_{1 s}=\left(2.91_{-0.43}^{+0.24}\right) \times 10^{-15} \mathrm{~S}$
DIRAC $\quad \longleftrightarrow\left|a_{0}-a_{2}\right| m_{+}=0.264_{-0.011}^{+0.020}$
NA48/2 : $\left(a_{0}-a_{2}\right) m_{+}=0.261 \pm 0.006 \pm 0.003 \pm 0.0013 \pm 0.013$ stat. syst. ext. theor.

Very little theoretical uncertainty in the prediction of the pionium lifetime because the interaction responsible for $\pi^{+} \pi^{-} \rightarrow \pi^{\circ} \pi^{\circ}$ is made effectively "weak" by the large pionium radius:
$R_{\text {pionium }} \approx R_{\infty} \frac{2 m_{e}}{m_{+}} \approx 3.9 \times 10^{-11} \mathrm{~cm} \quad\left(R_{\infty}:\right.$ Bohr radius for $\left.\mathrm{M}_{\text {nucleus }}=\infty\right)$
$R_{\text {pionium }} \gg$ strong interaction radius $\left(\sim 10^{-13} \mathrm{~cm}\right)$
$\Rightarrow$ very little overlap of the $\pi^{+} \pi^{-}$atomic wave function with the strong interaction volume

NA48/2 (PRELIMINARY): from ( $\mathrm{aO}-\mathrm{a} 2$ ) and a2 extract a0
(must take into account the statistical error correlation coefficient $\approx-0.92$ )

$$
a_{0} m_{+}=\underset{\text { stat. }}{0.224} \pm \underset{\text { svst. }}{0.008} \underset{\text { ext. }}{0.006} \pm \underset{\text { theor. }}{0.0013}
$$



The yellow area represents theoretical uncertainty (assumed Gaussian)
The dashed bars represent the theoretical uncertainty

Spares:Vus

## Form factors

Events generated according to the Dalitz plot density distribution

$$
\rho^{0}\left(E_{l}^{*}, E_{\pi}^{*}\right)=\frac{d N^{2}\left(E_{l}^{*}, E_{\pi}^{*}\right)}{d E_{l}^{*} d E_{\pi}^{*}} \propto A f_{+}^{2}(t)+B f_{+}(t) f_{-}(t)+C f_{-}^{2}(t)
$$

$A, B$ and $C$ are kinematic terms, and $t$ is the transferred 4-momentum to the lepton pair ( $q^{2}$ )

$$
f_{0}(t)=f_{+}(t)+\frac{t}{\left(m_{K}^{2}-m_{\pi}^{2}\right)} f_{-}(t)
$$

Use PDG 2006 form factors for Charge Kaon decays
Quadratic $f_{+}(t)=f_{+}(0)\left(1+\lambda_{+}^{\prime} \frac{t}{m_{\pi^{ \pm}}^{2}}+\frac{1}{2} \lambda_{+}^{\prime \prime} \frac{t^{2}}{m_{\pi^{ \pm}}^{4}}\right)$


Linear

$$
\begin{aligned}
& \lambda_{+}^{\prime}=0.02485 \pm 0.00163 \pm 0.00034 \\
& \lambda_{+}^{\prime \prime}=0.00192 \pm 0.00062 \pm 0.0071
\end{aligned}
$$

$$
\begin{aligned}
f_{0}(t) & =f_{+}(0)\left(1+\lambda_{0} \frac{t}{m_{\pi^{ \pm}}^{2}}\right) \\
& \lambda_{0}=0.0196 \pm 0.0012
\end{aligned}
$$

Other models considered - Pole $\quad f_{+, 0}(t)=f_{+}(0)\left(\frac{m_{V, S}^{2}}{m_{V, S}^{2}-t}\right)$

CP violation

## NA48: $\eta_{+-}$

$$
\underline{K}_{L} \rightarrow \pi^{+} \pi^{-}
$$

- Need to suppress main decay channels by 4-5 orders of magnitude
- Only small background of $\sim 0.5 \%$
- Data are well described by MC
- About 47000 selected $\pi^{+} \pi^{-}$events



## NA48: $\eta_{+-}$

Parameter $\eta_{+-}=$fundamental observable of CP violation, defined as the CP-violating ratio of the neutral kaon decaying into two charged pions

$$
\begin{aligned}
& \eta_{+-}:=\frac{A\left(K_{L} \rightarrow \pi^{+} \pi^{-}\right)}{A\left(K_{S} \rightarrow \pi^{+} \pi^{-}\right)} \quad \eta_{+-}=\epsilon+\epsilon^{\prime} \\
& \frac{\Gamma\left(K_{L} \rightarrow \pi^{+} \pi^{-}\right)}{\Gamma\left(K_{L} \rightarrow \pi^{ \pm} \mp{ }^{\mp} \nu\right)}=\left(4.835 \pm 0.022_{\text {stat. }} \pm 0.016_{\text {syst. }}\right) \times 10^{-3} \\
& =(4.835 \pm 0.027) \times 10^{-3} \\
& B R\left(K_{L} \rightarrow \pi^{+} \pi^{-}\right)=\frac{\Gamma\left(K_{L} \rightarrow \pi^{+} \pi^{-}\right)}{\Gamma\left(K_{L} \rightarrow \pi e \nu\right)} \cdot B R\left(K_{L} \rightarrow \pi e \nu\right) \\
& =(1,941 \pm 0.019) \times 10^{-3} \\
& \left|\eta_{+-}\right|=\sqrt{\frac{\tau_{K S}}{\tau_{K L}} \cdot \frac{B R\left(K_{L} \rightarrow \pi^{+} \pi^{-}\right)}{B R\left(K_{S} \rightarrow \pi^{+} \pi^{-}\right)}}=(2.223 \pm 0.012) \times 10^{-3}
\end{aligned}
$$





## NA48: $A_{g}$



$$
R(u)=\frac{N^{+}(u)}{N^{-}(u)} \sim 1+\frac{\Delta g u}{1+g u+h u^{2}}
$$

$$
A_{g}=\frac{g^{+}-g^{-}}{g^{+}+g^{-}} \approx \frac{\Delta g}{2 g}
$$



$$
\begin{aligned}
& A_{g}=\left(1.8 \pm 1.7_{\text {stat }} \pm 0.9_{\text {syst }}\right) \cdot 10^{-4} \\
& A_{g}=(1.8 \pm 1.9) \cdot 10^{-4}
\end{aligned}
$$

| $K^{ \pm} \rightarrow \pi^{ \pm} \pi^{0} \pi^{0}-$ <br> Smith ot al. (1975) at CERN-PS |
| :---: |
| TNF (2005) at IHEP Protvino |

