Topic 2: Imaging Properties
Workshop Solutions

Workshop Questions

2.1 Fourier Transform Theory

See tutorial questions and solutions in *The Fourier Transform, (What you need to know)*. Qustions 4, 7, 8 and 10 are particularly useful.

Solution

See solutions in Fourier Booklet.

2.2 Linear Imaging

State all the conditions required of an imaging system for it be be described by

$$f(x,y) = h(x,y) \odot o(x,y)$$

where $h(x,y)$ is the intensity point spread function.

Solution

To be described by the convolution integral the imaging systems must be:

1. **Linear:** so that the image con be assumed to be a linear summation of it constituent parts. This also assumes that the *object* is two dimensional.
2. **Space Invariant:** so that the imaging properties are identical at all positions.
3. **Isoplanatic:** so that a simple shift in the object results in a simple shift in the image (scaled by the magnification of the optical system.)
4. **Incoherent Illumination:** so that the resultant image is the sum of the *intensity* PSF.

2.3 Use of the OTF

Optical imaging theory shows that the OTF of an ideal lens is given by:

$$H(w) = \frac{2}{\pi}\left[\cos^{-1}\left(\frac{w}{w_0}\right) - \frac{w}{w_0}\sqrt{1-\frac{w^2}{w_0^2}}\right]$$

where $w^2 = u^2 + v^2$, and $w_0 = 1/\lambda \mathrm{F}_{\mathrm{No}}$. If for an $\mathrm{F}_{\mathrm{No}} = 8$ imaging system using green light, ideal image of the object in the image plane is

$$o(x,y) = 1 + \cos(2\pi a x)$$

calculate the detected image $f(x,y)$ when (i) $a = 100\mathrm{mm}^{-1}$, (ii) $a = 200\mathrm{mm}^{-1}$, (iii) $a = 300\mathrm{mm}^{-1}$

Solution

For a linear system, then

$$f(x,y) = h(x,y) \otimes o(x,y)$$

So in Fourier Space we have that:

$$F(u,v) = H(u,v)\,O(u,v)$$

We know the expression for $H(u,v)$ so to get $F(u,v)$ and hence $f(x,y)$ we need to calculate $O(u,v)$, which we can get from the shifting theorem to be:

$$\begin{aligned} O(u,v) &= \mathcal{F}\left\{1+\cos(2\pi ax\right\} \\ &= \mathcal{F}\left\{1\right\} + \frac{1}{2}\mathcal{F}\left\{\exp(\imath 2\pi ax)\right\} + \frac{1}{2}\mathcal{F}\left\{\exp(-\imath 2\pi ax)\right\} \\ &= \delta(u) + \frac{1}{2}\delta(u-a) + \frac{1}{2}\delta(u-a) \end{aligned}$$

Add in the effect of the OTF, which is just a multiplication, then, noting that $H(0) = 1$ and that $H(a) = H(-a)$ then,

$$F(u,v) = \delta(u) + H(a)\left[\frac{1}{2}\delta(u-a) + \frac{1}{2}\delta(u-a)\right]$$

we can now inverse Fourier Transform this to give

$$f(x,y) = 1 + H(a)\cos(2\pi ax)$$

which is the same for as the object but the contrast has been modified by the OTF. (The OTF is just the fidelity with which a grating with a particular frequency is passed by the optical system.)

Numerical examples: for visible light $\lambda = 550\,\mathrm{nm}$ so that

$$w_0 = 227\mathrm{mm}^{-1}$$

For $a = 100\mathrm{mm}^{-1}$ then $H(100) = 0.719$, so that

$$f(x,y) = 1 + 0.719\cos(2\pi ax)$$

For $a = 200\mathrm{mm}^{-1}$ then $H(200) = 0.076$, so that,

$$f(x,y) = 1 + 0.076\cos(2\pi ax)$$

so much lower contrast.

For $a = 300\mathrm{mm}^{-1}$ then $H(300) = 0$, so that,

$$f(x,y) = 1$$

so constant. No information about the grating is passed since it is above the bandwidth of the optical system.

2.4 The Spot Satellite

The *Spot* satellite has a ground resolution in the near infra-red (at 1.3μm) of approximately 10 m from an orbit of 832 km. Assuming that this resolution limit is due to the point spread function of the imaging telescope estimate its diameter.

If the CCD sensors has a pixel size of $10 \times 10\mu$m, estimate the focal length and diameter of the optical system.

Look up the technical information on the Spot satellite system, and see how close there estimates are.

Solution

If the satellite has a ground resolution of 10 m from and orbit of 832 km then it has *angular* resolution of

$$\theta_0 = \frac{10}{832 \times 10^3} = 1.20 \times 10^{-5}$$

The angular resolution of a telescope of diameter d is given by

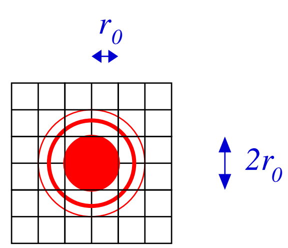
$$\theta_0 = \frac{1.22\lambda}{d} \quad \Rightarrow \quad d = \frac{1.22\lambda}{\theta_0}$$

so for visible light, $\lambda = 550\,\mathrm{nm}$ then the diameter of the telescope is 132 mm.

The radius of the point spread function is therefore,

$$r_0 = \frac{1.22\lambda f}{d} = f\theta_0$$

The pixel size should be smaller than the total size of the PSF which is $2r_0 \times 2r_0$.

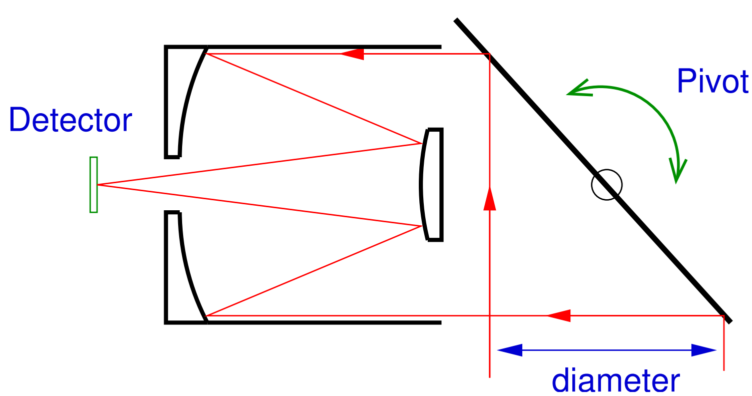


A reasonable scheme would be to have pixel size of 10μm matched to r_0, so that

$$f = \frac{10\mu\mathrm{m}}{1.2 \times 10^{-5}} = 833\mathrm{mm}$$

which gives the telescope a $\mathrm{F_{No}} = 6.3$.

This is a reasonable system for a satellite, typically this is a twin mirror telescope of the type:



In a practical system the telescope would have somewhat wider aperture to allow more light into the system.