Topic 7: Digital reconstruction
Workshop Solutions

Workshop Questions

7.1 Shape of $H(k,l)$ for linear blur

For a 256 by 256 image convolved with a linear blur with a length of M pixels, calculate the analytical expression for $H(k,l)$. Sketch this function and discuss how the first zero is located to the length of the linear blur.

Solution

The linear blur PSF of length M pixels is just a rectangle of length M and width 0. From question 2 in the FOURIER booklet we have that the analytic Fourier Transform of a rectangular of size $a \times b$ is given by:

$$H(u,v) = \text{sinc}(\pi au)\text{sinc}(\pi bv)$$

where $\text{sinc}(x) = \sin(x)/x$.

If in real space we take the sampling interval $\Delta x = 1$, then for an image of size $N \times N$ pixels the sampling interval in Fourier space is $\Delta u = 1/N$. So the digital of a linear blur of M pixels is given by
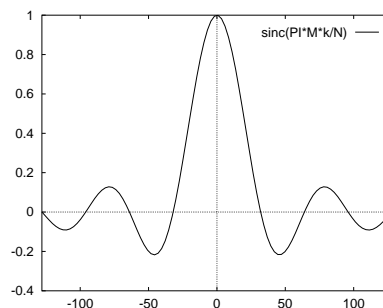
$$H(k,l) = \text{sinc}\left(\frac{\pi M}{N}k\right)$$

which is a one-dimensional sinc in the k direction with the first zero at

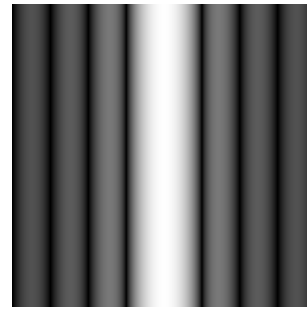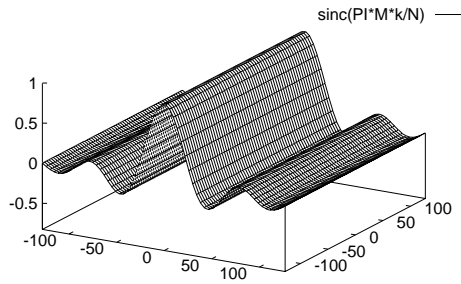$$\frac{\pi M}{N}k_0 = \pi \quad \Rightarrow \quad k_0 = \frac{N}{M}$$

so that the "width" of the sinc is inversely proportional to the length of the blur.

Example: for a 256×256 image with a blur of length 8 pixels then $H(k,0)$ is given by:

with the first zero at ± 32 as expected.

In two dimensions the $H(k,l)$ we get

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where the image is from the `fourier` program, and is thus an intensity plot of $\log\left(|H(k,l)|^2 + 1\right)$.

7.2 Wiener Filter Simulation

Try the `wiener` filter simulation programme to experiment with the effect of noise levels and SNR parameter in the Wiener Filter.

The program is based on linear horizontal blur PSF and operates as follows:

1. Initially asked for an input "ideal" image. The `toucan.pgm` is a good start.

2. Asks for size of horizontal blur in pixels. Blur function is a horizontal line of the specified length, 1 pixel wide. (Try 10 to 20 pixels).

3. Optionally adds Gaussian signal dependent noise to give a specified SNR. The added noise has standard deviation,

$$\sigma_n = \frac{\sigma_f}{SNR}$$

where σ_f is calculated from the input image.

4. The output image is then displayed. (due to buffering they may be delayed).

5. The SNR used in the wiener filter is then asked for. Note: Your should experiment by changing the SNR to values other than that used in the noise, in particular what happens if the wiener SNR is too high?

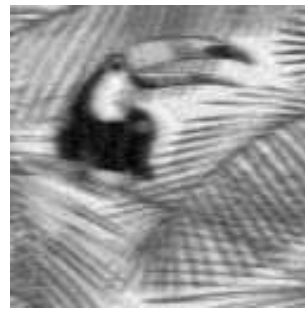6. The reconstruction is then displayed

Solution

You should try this for yourself. Here are typical results for

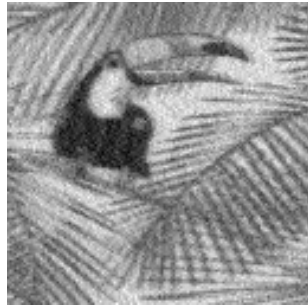|	
Image Size	128
Blur Length	10
SNR (added noise)	10

This image was then deconvolved using the wiener filter with values of SNR or 4,10,100. Results shown below:
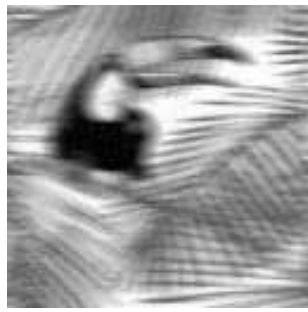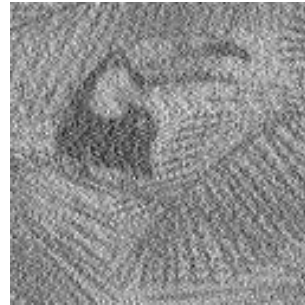
Blurred Image

SNR $= 4$ Reconstruction

SNR $= 10$ Reconstruction

SNR $= 100$ Reconstruction

These reconstructions show,

1. Low estimate for SNR (4 in this case) gives a reasonable reconstruction, but is rather "low-pass" filtered resulting in loss of high frequency detail such as the sharp edges in the palm leaves.

2. The medium estimate of SNR (10) gives less "low-pass" appearance but there is evidence of noise resulting is a rather "blotchy" effect in smooth areas. This is a typical problem with the wiener filter.

3. A high estimate for SNR results is sever amplification of the noise with tend to "swamp" out the reconstruction.

There is always this trade-off and the "best image" is usually a compromise between noise and low-pass effects. In practice SNR terms in the wiener filter should be used as a variable parameter and the SNR calculated from the image only used to give an initial estimate. The optimal value has typically to be found by "trial and error".