Topic 10:

Applications of Holography

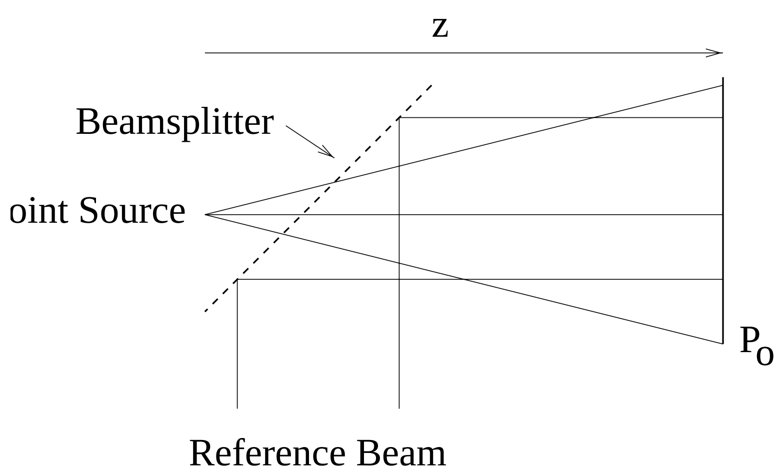
Aim: To review a range of applications of holography, including holographic lenses and commercial holographic applications.

Contents:

- Holographic Lens
- Variations on Holographic Lens
- Hologon
- Head-Up display

Holographic Lens

Consider a hologram made with a plane beam reference and a point source object.



Then at P_0 the object wave is

$$o(x,y)\exp(\imath\Phi(x,y)) = Ah(x,y;z)$$

where $h(x,y;z)$ is the *Free Space Response Function*.

The reference wave is perpendicular to the plate, so is just

$$r\exp(\Phi_0) = \text{Constant}$$

Taking the Fresnel Approximation we have that

$$h(x,y;z) = \frac{\lambda\exp(\imath\kappa z)}{\imath z}\exp\left[\imath\frac{\kappa}{2z}(x^2+y^2)\right]$$

so that the object wave is,

$$o_0\exp\left[\imath\kappa\left(z+\frac{(x^2+y^2)}{2z}\right)\right]$$

where

$$o_0 = \frac{A\lambda}{\imath z}$$

The intensity in plane P_0 is then,

$$g(x,y) = \left| r\exp(\Phi_0) + O_0 \exp\left[\imath\kappa\left(z + \frac{(x^2+y^2)}{2z}\right)\right]\right|^2$$

which, with some re-arrangement, gives

$$r^2 + |O_0|^2 + 2rO_0\cos\left[\kappa\left(z+\frac{x^2+y^2}{2z}\right)\Phi_0\right]$$

Now if we assume that,

$$\kappa z = \Phi_0 \pm 2n\pi$$

then the intensity pattern can be simplified to give,

$$r^2 + |O_0|^2 + 2rO_0\cos\left(\kappa\frac{x^2+y^2}{2z}\right)$$

Which is the equation of a set of circular fringes with a bright fringe when

$$\frac{\kappa\rho^2}{2z} = 2n\pi \quad \text{where} \quad \rho^2 = x^2 + y^2$$

or the radius of the n^{th} fringe is given by

$$\rho_n = \sqrt{2n\lambda z}$$

This is Just Newton’s Rings

Holographic Reconstruction

Form hologram as previous with

$$g(x,y) = g_0\left(1 + \delta\hat{g}(x,y)\right)$$

with, in this case

$$\delta\hat{g}(x,y) = \frac{2rO_0}{g_0}\cos\left(\kappa\frac{x^2+y^2}{2z}\right)$$

Expose hologram and develop to get Amplitude Transmittance,

$$T_a = T_0 - a\delta\hat{g} + b(\delta\hat{g})^2$$

Reconstruct with Collimated Beam

$$u(x,y) = r\exp(\imath\Phi_0) \quad \text{Let } \Phi_0 = 0$$

then the transmitted amplitude is

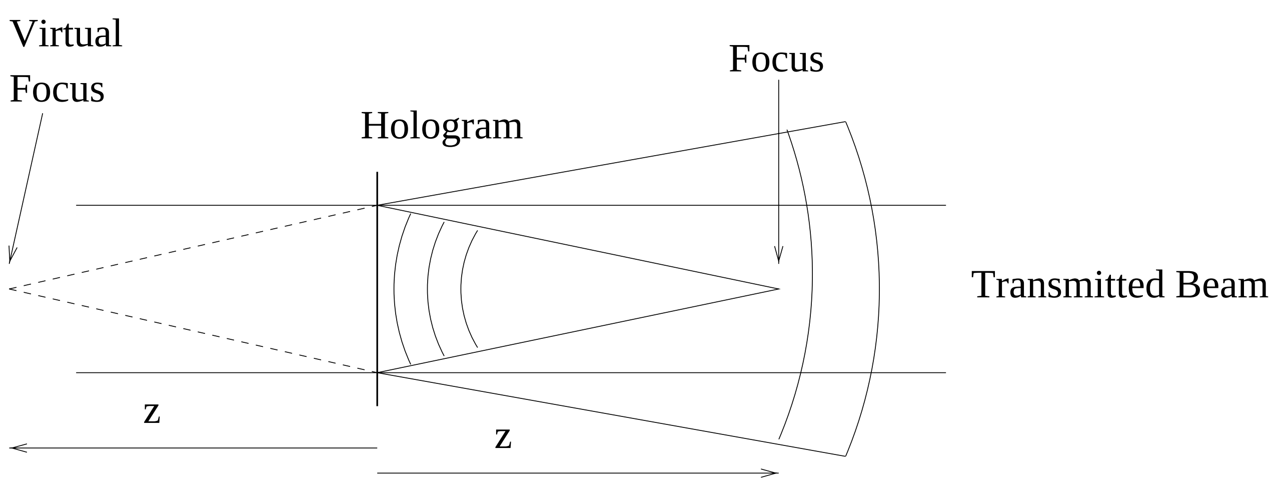
$$v(x,y) = rT_a$$

1) First Two Terms Let $b = 0$
We can expand to get **Three** terms,

$$\begin{aligned} v(x,y) &= rT_0 + && (1)\\ &\quad \frac{ar^2O_0}{g_0}\exp\left(-\imath\frac{\kappa}{2z}(x^2+y^2)\right) + && (2)\\ &\quad \frac{ar^2O_0}{g_0}\exp\left(\imath\frac{\kappa}{2z}(x^2+y^2)\right) && (3)\end{aligned}$$

which give

1. Partially transmitted DC beam
2. Lens of focal length z
3. Lens of focal length $-z$



Very similar to a Zone Plate.

See tutorial on Zone Plate for comparison.

2) Add non-linear Term $b \neq 0$
Then we get **Three** extra terms,

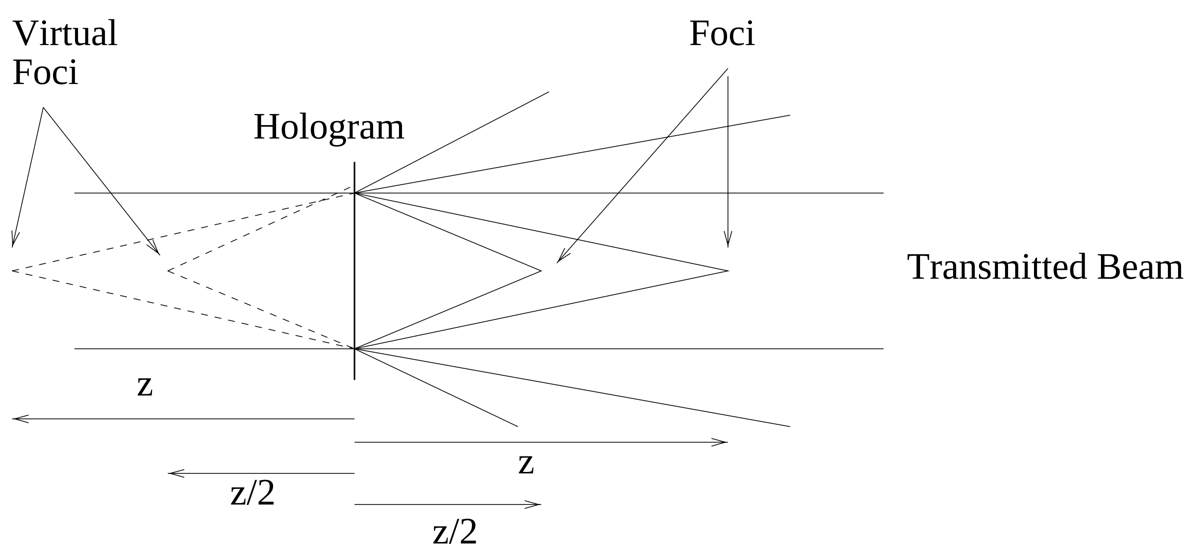
$$\frac{br^3 O_0^2}{g_0^2} + \tag{4}$$

$$\frac{br^3 O_0^2}{g_o^2} \exp\left(-\imath \frac{\kappa}{z}(x^2+y^2)\right) \tag{5}$$

$$= \quad \frac{br^3 O_0^2}{g_0^2} \exp\left(\imath \frac{\kappa}{z}(x^2+y^2)\right) \tag{6}$$

which give

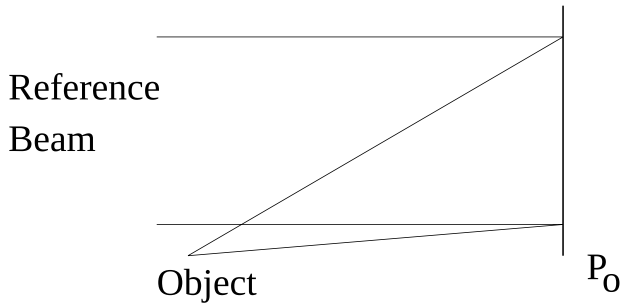
1. Extra transmitted DC beam.
2. Lens of focal length $z/2$
3. Lens of focal length $-z/2$



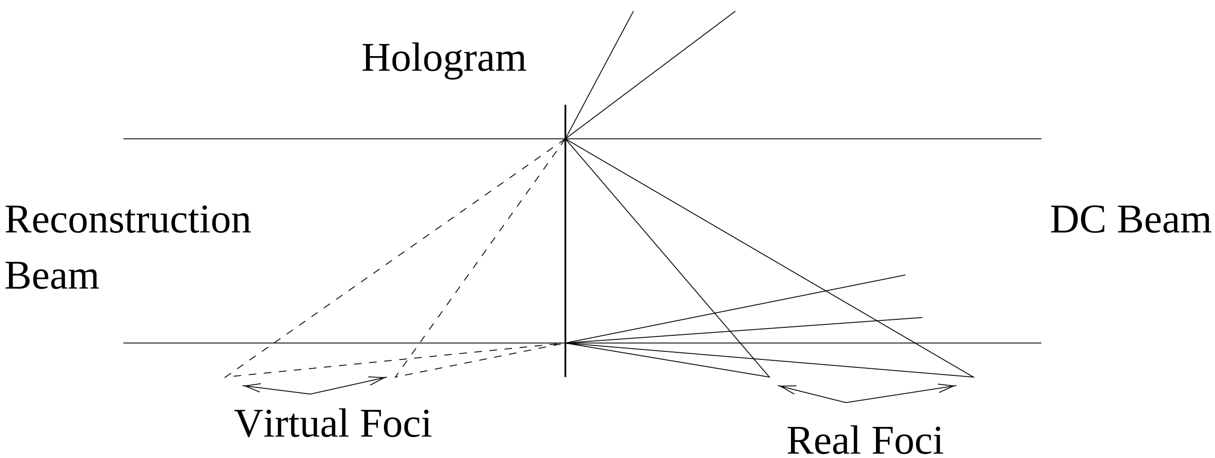
Problem with overlapping orders

Variations of Holographic Lens.

Change the formation geometry, (move off-axis).



When we reconstruct we then get,



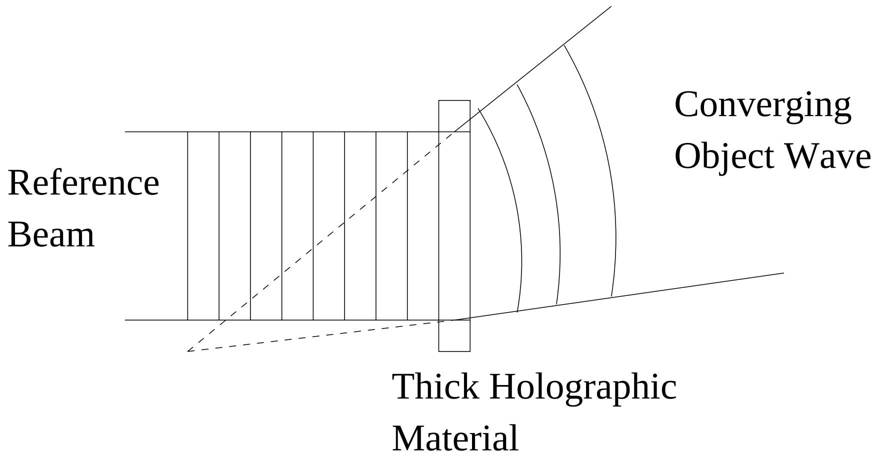
With the order separated, and useful foci (both real and virtual).

Note with holographic lenses.

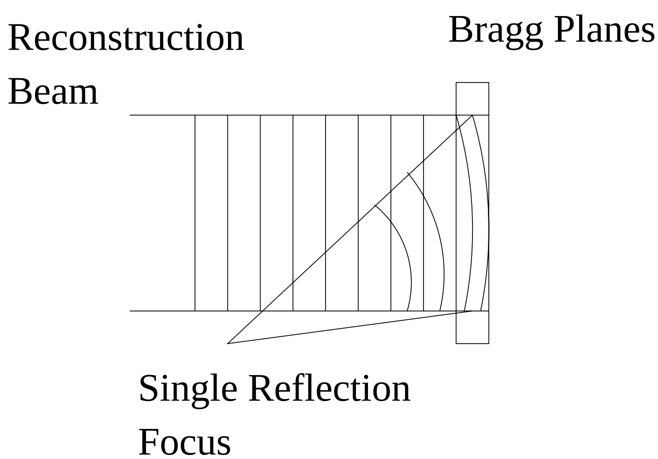
- Reconstruction work equally well in Kirchhoff region, so able to form very wide aperture systems.
- Bleach lenses to get diffraction efficiency of $\approx 33\%$
- Only works in monochromatic light

Thick Holographic Lenses

Make reflection, Thick Hologram with object and reference beam from] opposite sides



When we reconstruct we then get,



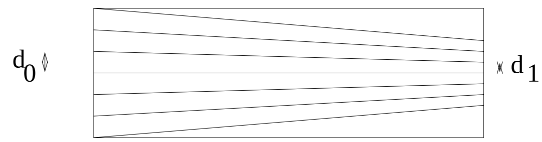
We can get single focus with efficiency up to 90%.

Very useful technique to produce wide aperture lenses for compact optical systems, (still monochromatic ONLY).

Major Potential:
Make holographic lens in red light, then swell gelatin to reconstruct in the Infra-Red where it is difficult to make lenses. (considerable potential)

Practical Systems.

1) The Hologon:
Consider forming a grating of the type,

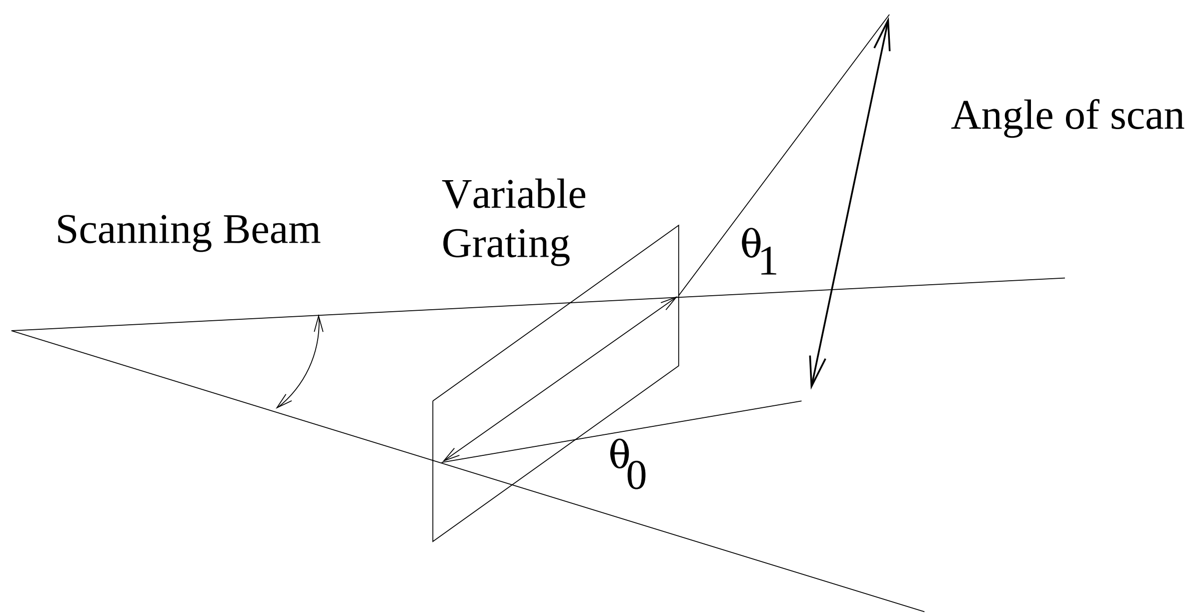


Then the diffraction angle will be given by

$$\sin\theta = \frac{\lambda}{d}$$

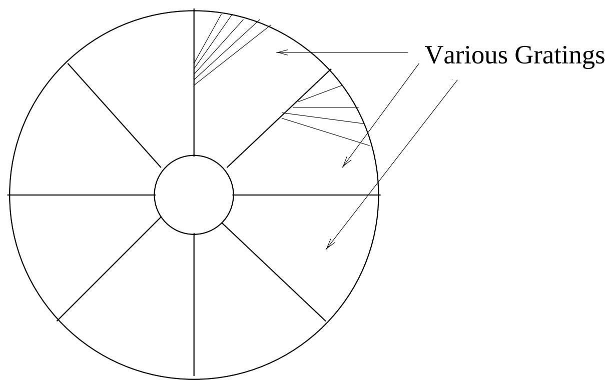
where d varies between $d_0 \rightarrow d_1$ across the grating.

Illuminate this with a scanning beam,

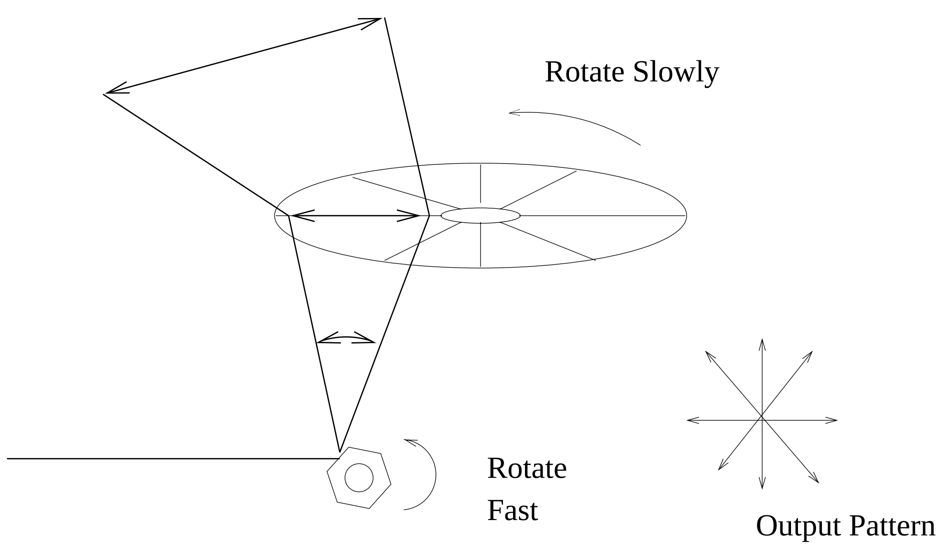


Angle of the diffracted scan is given by the grating spacing variation.

Make “wheel” containing a range of these gratings at different angles,

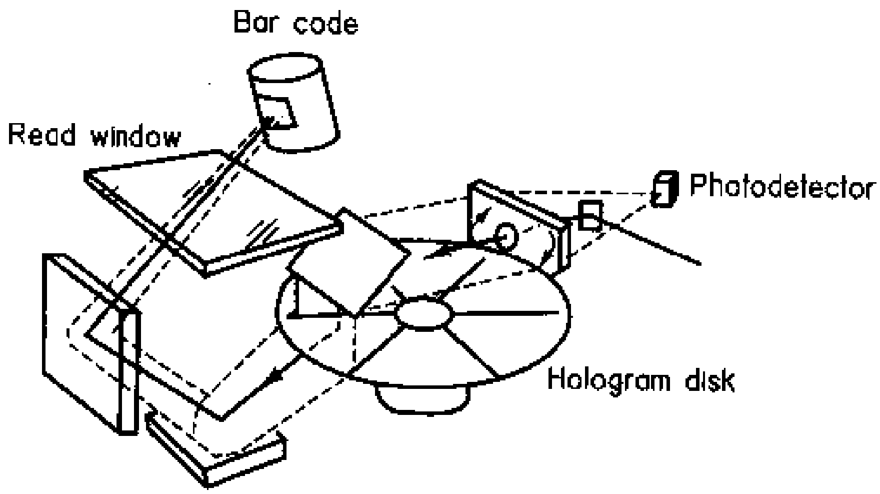


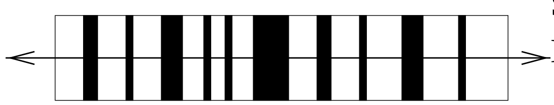
Then assemble the whole systems as



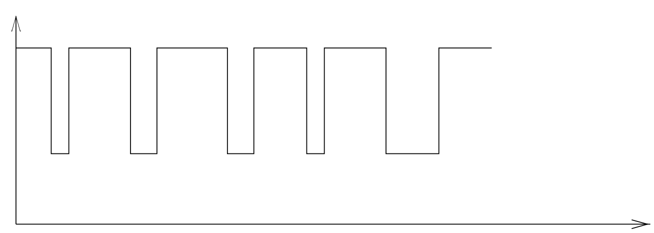
The output scan speed is determined by the speed of the rotating hexagonal prism and the orientation by the sector of the Hologon.

System is the basis of the Automatic Bar-Code reader.





Measure the **Reflected Light**, and we get



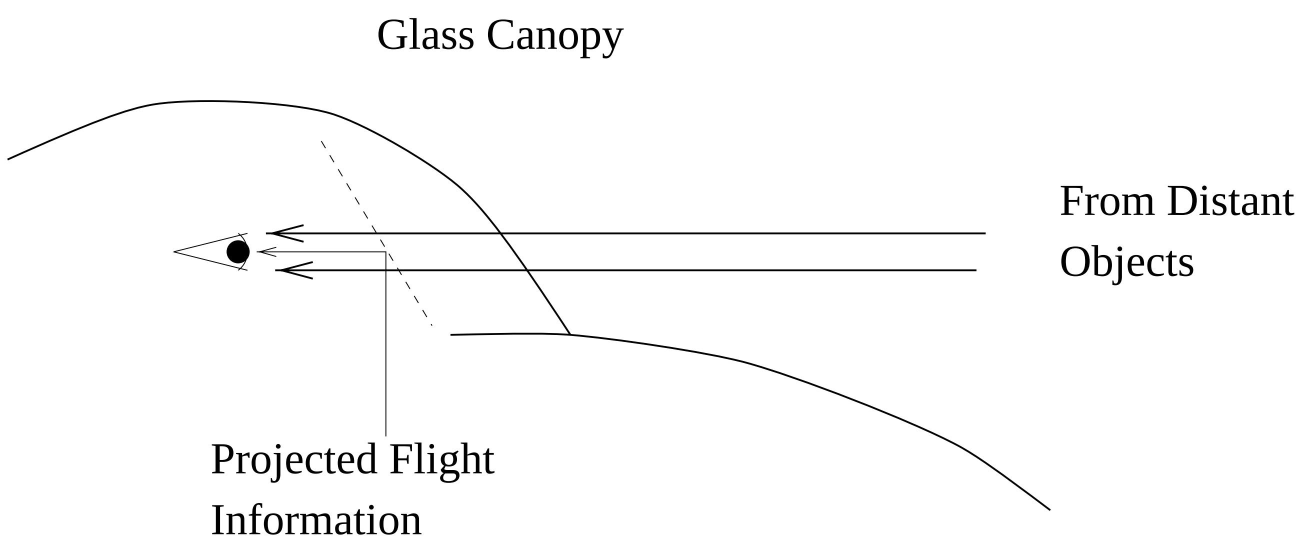
Reflected light then analysed to “read” the 12 digit bar-code.

The largest use of holograms

Actual holograms and optics make-up a significant part of the cost of a supermarket check-out.

Head-Up Displays

Used mainly in military aircraft as in-flight projection system.



Aim: Project instruments (and target) information into pilots field of view.

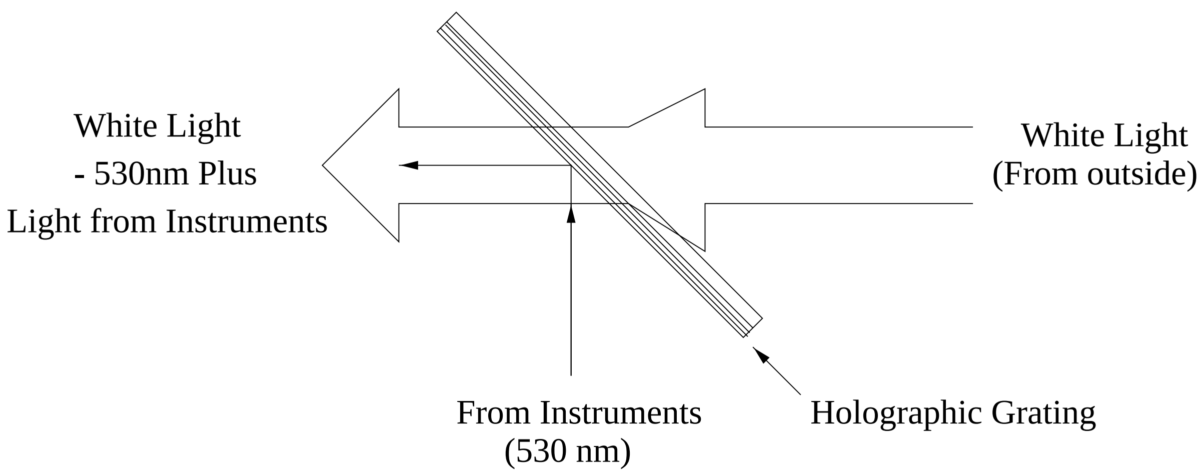
1): Semi-Silvered Mirror:
Old system, with many problems,

1. Light loss from distant objects
2. Reflection too dim
3. Reflections from inside canopy
4. Very small angle of view, and sever distortions

2): Holographic Filter:
Use volume hologram to reflect at wavelength to match instruments.

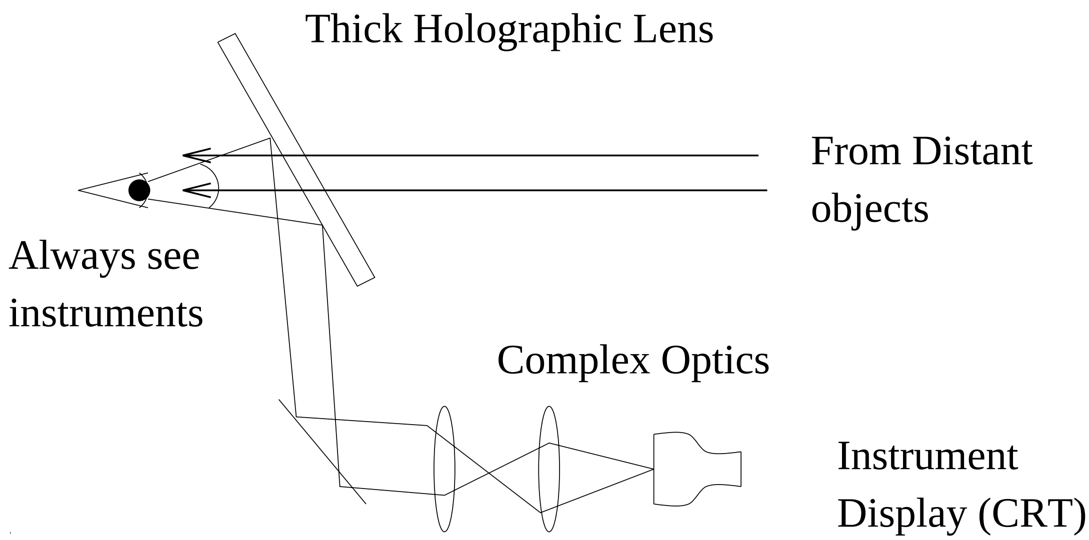
Typical filter:

- **External Transmittance** 97% except about selective wavelength. (5nm region lost)
- **Instrument Illumination** 95% reflectivity about selected wavelength (typically 530nm).



This "flat" holographic filter removes the light efficiency problems, but not the movement problems.

3) Holographic Lens:
Make a thick holographic lens that operates like the holographic filter, but **also** as an imaging lens.



By making filter contain a lens, able to project the instruments to appear at infinity, and also make them visible over a “reasonable” range of angles.

Works well in military aircraft since,

- Small view angle needed
- Most pilots the same size, (head in the same place).
- Able to use very expensive optics

Head-Up displays being worked-on for civilian aircraft and cars. Car prototypes with a range of manufactures, but still a good few years off.