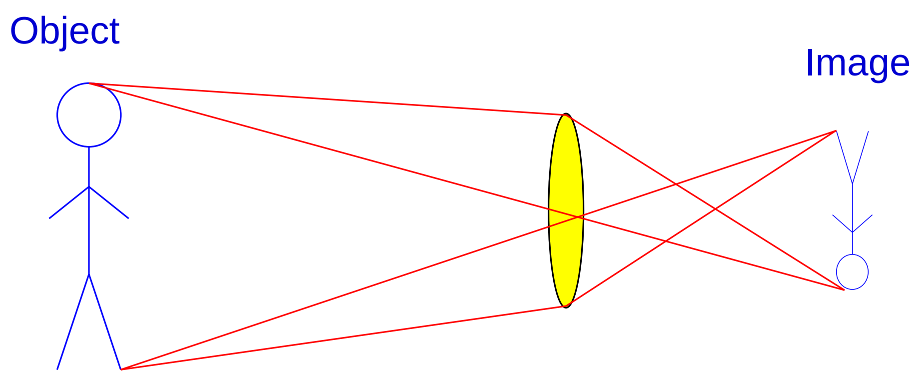
Topic 8: Holography

Aim: To cover the basic of holographic recording and reconstruction and review holographic materials.

Contents:

Photography

Record optical distribution as **Optical Density** given by **intensity only**.



$$D(x,y) = \gamma \log_{10}(E(x,y)) - D_0$$

where

$$E(x,y) = \tau |u(x,y)|^2$$

Do not record the **Phase Information**, so

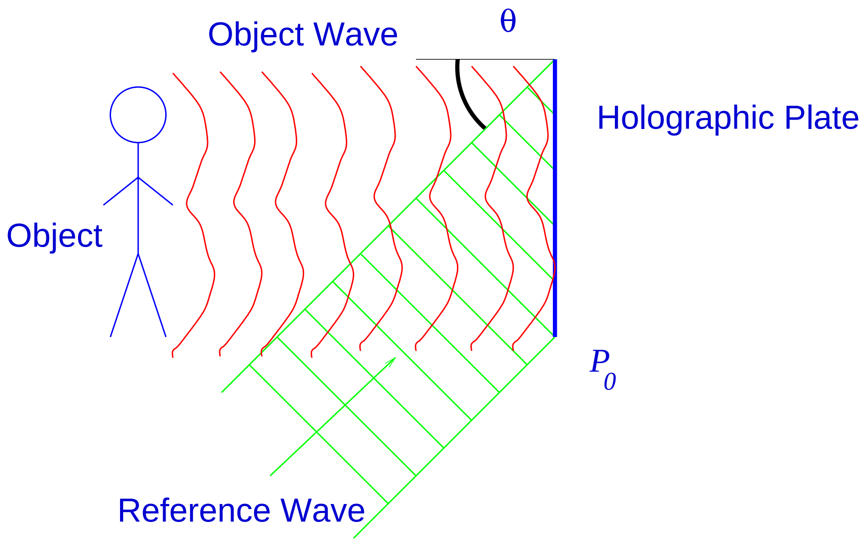
- No depth information
- Two dimensional projection of three dimensional scene.
- Similar for coherent and incoherent, (different transfer function)

We have to do something different to retain phase information.

Basic Holography

To retain phase information we must **encode** complex distribution as intensity pattern.

Encode by adding reference beam:



At P_0 we have **two** optical distributions

$$o(x,y)\exp(\imath\Phi(x,y)) \rightarrow \text{Scattered from object}$$
$$r\exp(\imath\kappa x\sin\theta) \rightarrow \text{Reference Wave}$$

where r is a constant and θ is angle from plate normal

Assume that the beams are **coherent**, then Amplitudes add to give,

$$u(x,y) = r\exp(\imath\kappa x\sin\theta) + o(x,y)\exp(\imath\Phi(x,y))$$

Intensity in P_0 is given by

$$g(x,y) = |u(x,y)|^2$$

which after some expansion is given by,

$$g(x,y) = |r|^2 + |o(x,y)|^2 + 2ro(x,y)\cos(\kappa x\sin\theta - \Phi(x,y))$$

There is **no image**, so for all practical cases:

$$o(x,y) \rightarrow \text{Varies slowly over } (x,y)$$

so we can assume that

$$|r|^2 + |o(x,y)|^2 \approx \text{constant}$$

but we have that:

$$\theta \rightarrow \textbf{NOT} \text{ small}$$

the intensity can be written as:

$$g(x,y) = g_0 + 2ro(x,y)\cos\left(\kappa x \sin\theta - \Phi(x,y)\right)$$

which is high frequency $\cos()$ fringes in plane P_0

- **Amplitude of fringes** encodes $o(x,y)$
- **Location of fringes** encodes $\Phi(x,y)$

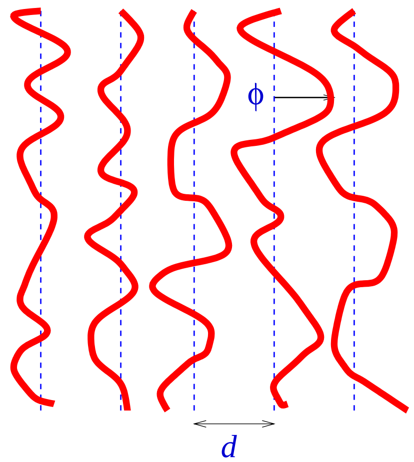
We have encoded both the Amplitude and the Phase of the object wave $o(x,y)$ as an **intensity** distribution.

Shape of Fringes

Maxima of intensity when

$$\kappa x \sin\theta - \Phi(x,y) = 2n\pi$$

so if θ large, then $\Phi(x,y)$ displaces fringes from regular pattern



If $\Phi(x,y)$ is a random variable, then mean separation

$$d = \frac{\lambda}{\sin\theta}$$

Example: $\theta = 30^\circ$, and $\lambda = 633$nm (He-Ne) then

$$d = 2\lambda \approx 1.3\mu\text{m}$$

or

700lines/mm High Frequency

Need a very high resolution photographic emulsion.

Fine Grain, very slow photographic material needed. (special photographic material)

Need to record the fringe locations, so need a higher resolution than this, 1200 lines/mm is typical.

Hologram Formation

Expose emulsion in the linear region and develop to form negative.

Amplitude Transmission is then:

$$T_a = 10^{D_0/2}(\tau g)^{-\gamma/2} = Kg(x,y)^{-\gamma/2}$$

we have that the intensity

$$g(x,y) = g_0 + 2ro(x,y)\cos(\kappa x \sin\theta - \Phi(x,y))$$

where we have assumed $|o(x,y)|^2$ is slow varying. This can be written as:

$$g(x,y) = g_0 + \delta g(x,y)$$

where we have that:

$$\delta g(x,y) = 2ro(x,y)\cos(\kappa x \sin\theta - \Phi(x,y))$$

This gives the Amplitude Transmission as

$$T_a = K(g_0 + \delta g)^{-\gamma/2}$$

which can then be written as

$$T_a = Kg_0^{-\gamma/2}\left(1 + \frac{\delta g}{g_0}\right)^{-\gamma/2} = Kg_0^{-\gamma/2}(1 + \delta\hat{g})^{-\gamma/2}$$

where

$$\delta\hat{g} = \frac{\delta g}{g_0}$$

Assume: that $g_0 \gg |\delta g(x,y)|$. (Assume low contrast fringes on a large background).

Expand the term, to second order to get;

$$(1+\delta\hat{g})^{-\gamma/2} = 1 - \frac{\gamma}{2}\delta\hat{g} + \frac{\gamma(\gamma+2)}{8}(\delta\hat{g})^2$$

Substituting this back into the expression the T_a we get

$$T_a = K g_0^{-\gamma/2}\left(1 - \frac{\gamma}{2}\delta\hat{g} + \frac{\gamma(\gamma+2)}{8}(\delta\hat{g})^2\right)$$

which we will write as:

$$T_a = T_0 - a\delta\hat{g} + b(\delta\hat{g})^2$$

where T_0, a and b are constants given by:

$$\begin{aligned} T_0 &= K\, g_0^{-\gamma/2} \\ a &= K\frac{\gamma}{2}\, g_0^{-\gamma/2} \\ b &= \frac{K\gamma(\gamma+2)}{8}\, g_0^{-\gamma/2} \end{aligned}$$

For most emulsions $\gamma \approx 1$ so $T_0 \approx a \approx b$, but

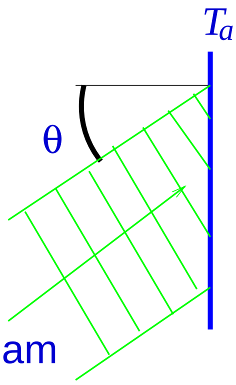
$$\delta\hat{g} \ll 1$$

so that

$$T_0 \gg |a\delta\hat{g}| \gg |b(\delta\hat{g})^2|$$

Reconstruction

Reconstruct with the original reference beam,



which is

$$u(x,y) = r\exp(\imath\kappa x\sin\theta)$$

The Complex Amplitude transmitted by the hologram is then

$$v(x,y) = T_a(x,y)\,u(x,y)$$

Look at First Two Terms: (assume $b = 0$),

$$v(x,y) = u(x,y)\,T_0 - u(x,y)\,a\delta\hat{g}(x,y)$$

which with substitution for $u(x,y)$ and $\delta\hat{g}$, gives

$$\begin{aligned} v(x,y) &= T_0 r\exp(\imath\kappa x\sin\theta) + \\ & ar\exp(\imath\kappa x\sin\theta)\frac{2ro(x,y)}{g_0}\cos(\kappa x\sin\theta - \Phi(x,y)) \end{aligned}$$

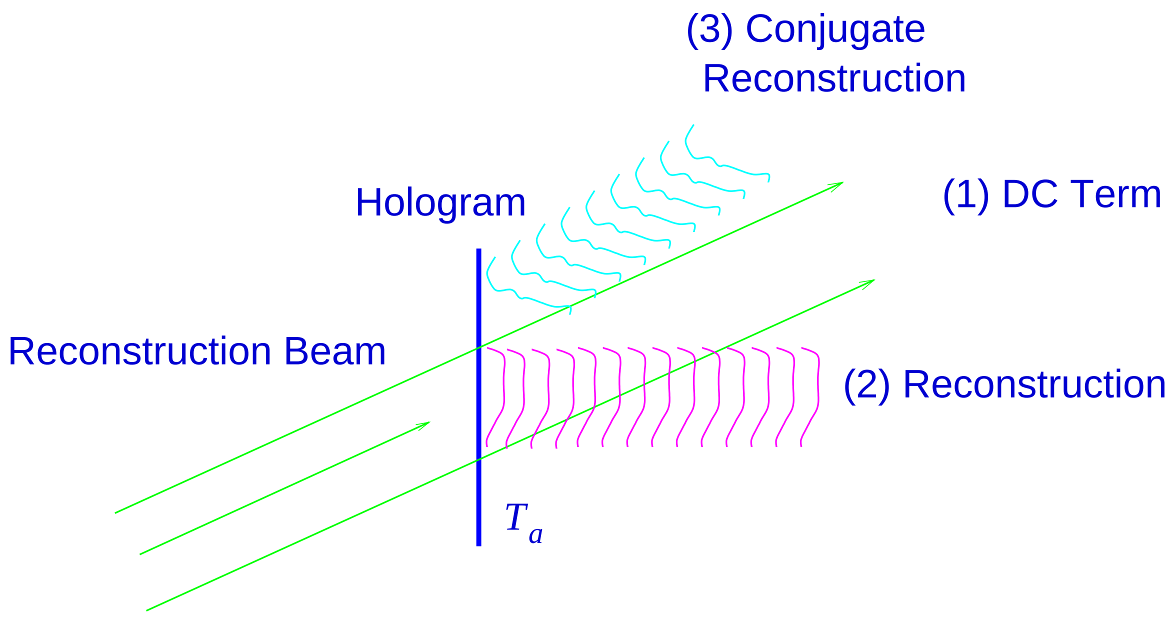
If we new expand the $\cos()$ term and cancel term, be get three terms

$$\begin{aligned} v(x,y) &= T_0 r\exp(\imath\kappa x\sin\theta) - & (1) \\ & \frac{ar^2}{g_0}o(x,y)\exp(\imath\Phi(x,y)) - & (2) \\ & \frac{ar^2}{g_0}o(x,y)\exp(-\imath\Phi(x,y))\exp(\imath 2\kappa x\sin\theta) & (3) \end{aligned}$$

Look at the three terms.

1. Partially transmitted reconstruction beam in direction θ.
2. **Reconstruction** of original complex object wave. Both amplitude and phase reconstructed. Note $-$ sign, which gives phase shift or π. (discussed later).
3. **Conjugate Reconstruction** Similar to Reconstruction, but complex conjugate. In direction ϕ where $\sin\phi = 2\sin\theta$

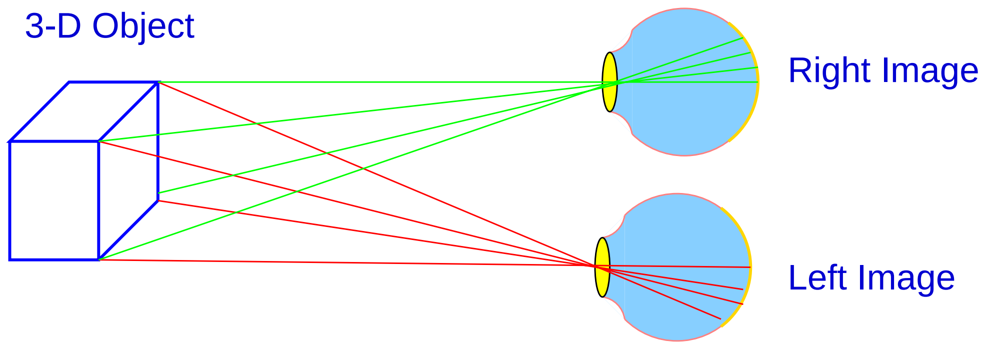
So provided that θ is **NOT** small, three terms will be separated.



Three terms separated. Only want (2) which is full three dimensional reconstruction of object wave.

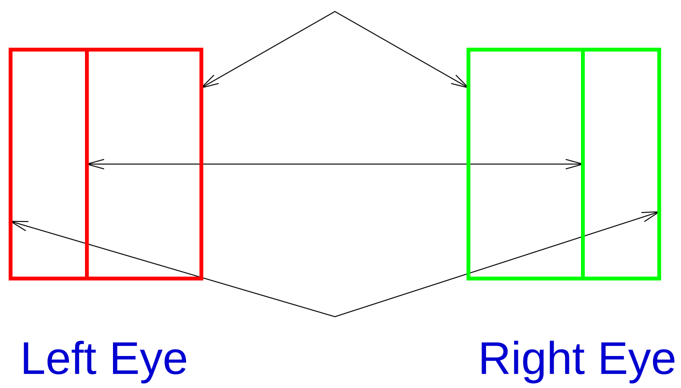
We "see in 3-D"

We live in a 3-D world, and we see in "3-D".



We have **two** eyes separated by about 65 mm.

We see two images of the same object from different directions,

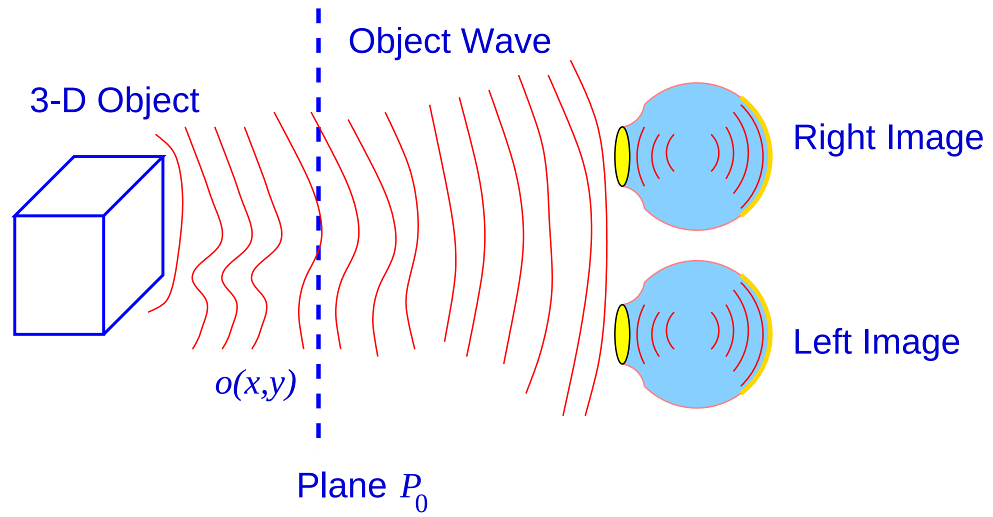


Brain "matches up" the vertical disparities and interperates the difference as "depth".

Because of our **two** eyes we can see in 3-D.

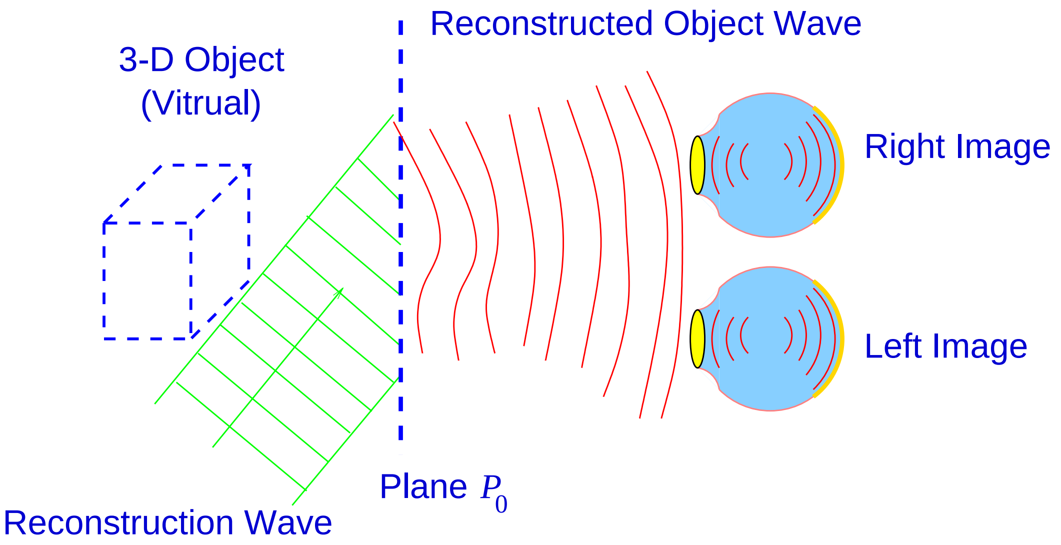
Scattered Light from Object

If we consider our wave model, then we have:



See two different images, and again the brain makes the 3-D scene.

Green**Record** *Amplitude distribution* in plane P_0, and *play-it-back*.



Reconstruct Amplitude Distribution in plane P_0 we will still see the two images, and hence a 3-D virtual image of the original object.

Non-Linear Terms

Add in the third term of

$$u(x,y)b\delta\hat{g}(x,y)^2$$

which by substituting for δf gives

$$r\exp(\imath\kappa x\sin\theta)b\left(\frac{2ro(x,y)}{g_0}\cos(\kappa x\sin\theta-\Phi(x,y))\right)^2$$

If we then expand the $\cos()$ term and collect terms, we get

$$\frac{2br^3}{g_0^2}o^2(x,y)\exp(\imath\kappa x\sin\theta)+ \quad (4)$$

$$\frac{br^3}{g_0^2}o^2(x,y)\exp(\imath 2\Phi)\exp(-\imath\kappa x\sin\theta)+ \quad (5)$$

$$\frac{br^3}{g_0^2}o^2(x,y)\exp(-\imath 2\Phi)\exp(3\imath\kappa x\sin\theta) \quad (6)$$

We get three additional terms,

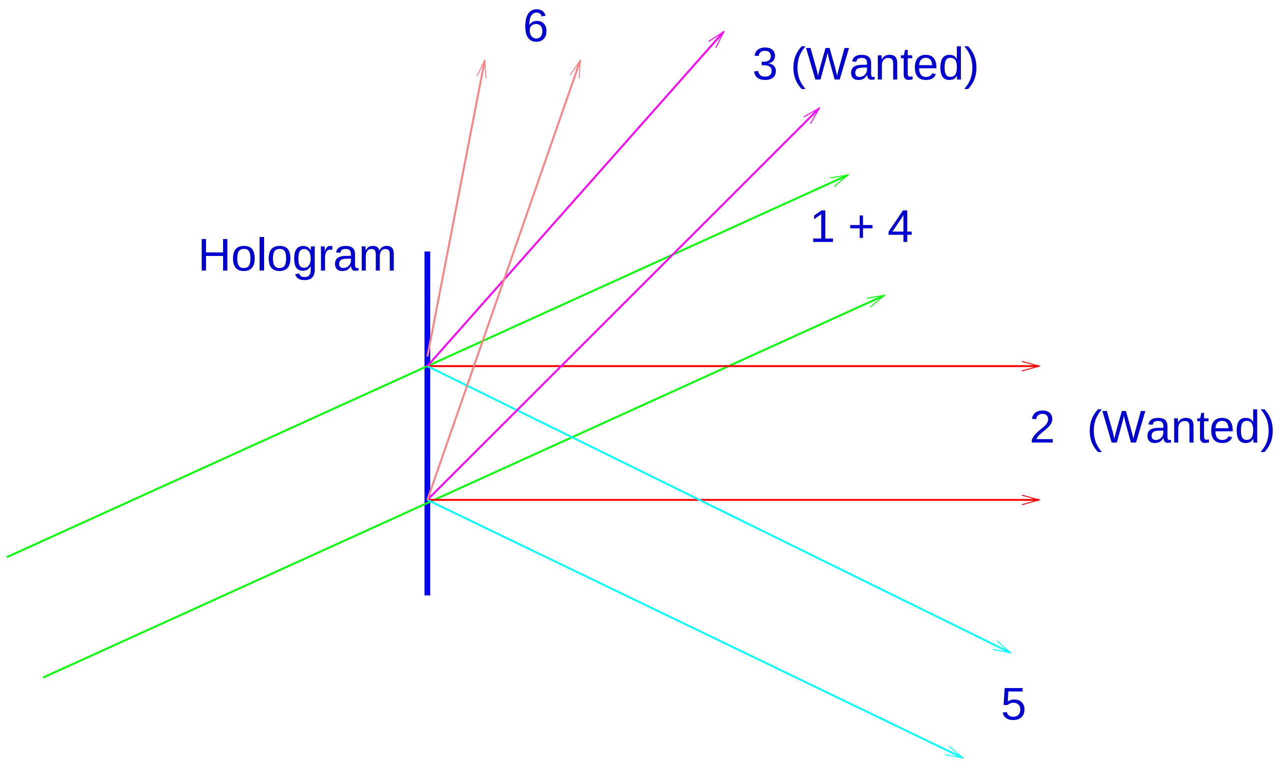
4 Additional transmitted term, (Note: $o^2(x,y)\approx$ constant.

5 Reconstruction of square of object wave, but in direction $-\theta$.

6 Reconstruction of square of Conjugate in direction ϕ, where $\sin\phi = 3\sin\theta$

With correct choice of θ none of these three additional terms will effect term (2) and (3) (the required reconstruction).

Full Reconstruction



Useful terms (2) and (3) separated from the other 4 unwanted terms.

Note if $\theta > 30^\circ$ then term 6 will be lost.

Holography is not effected by terms to second order.

Able to control the intensity of the second order terms by the changing ratio of $|r|^2$ to $|o(x,y)|^2$ during exposure. "It can be shown" that

$$\frac{I_2}{I_5} \approx \frac{16}{(\gamma+2)^2}\left[\frac{r^2}{o^2}+2\right]$$

(See tutorial)

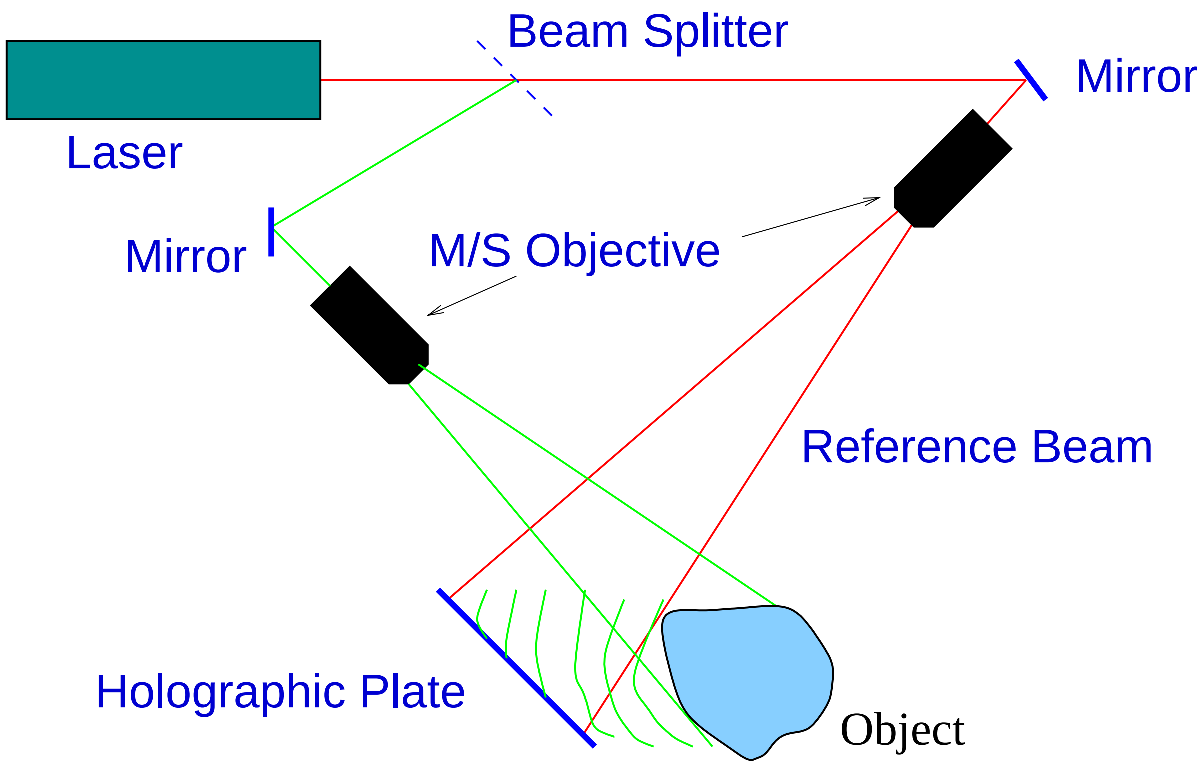
Practical System

Want all terms separated, so θ **not** small

$$\theta \approx 30^{\circ} \quad \text{typical}$$

Need **very** high resolution photographic material, (1200 line/mm typical)

Very fine grains, so **Very Slow**. so either long exposure **or** lots of light.



To get interference we need beam path to be approx. the same length.

Note: Reference beam not usually collimated. Mathematics are the same $\pm$ a few parabolic phase terms.

Low Power Holography

Use small CW laser (10mW) with 10cm plate. Typical exposure

$$\tau \approx 1\text{sec}$$

Major stability problem.

Fringe pattern **must** not move more that $1/4$ of fringe during exposure. All components must be stable to $\lambda/2$ or better.

Require solid table, mechanical isolation, stable temperature and minimal air currents.

High Power Holography

Same basic system, but use high powered pulsed laser, eg Ruby,

$$\tau \approx 5 \rightarrow 30\ \text{nsecs}$$

Able to make holograms of fast moving objects, (turbine blades, bullets, even people)

Stability **not** a problem, but **very** expensive, and safety a major issue.

Types of Holograms

1) Thin Amplitude Hologram

Expose holographic film to give Amplitude Transmittance

$$T_a = T_0 - a\delta\hat{g} + b\delta\hat{g}^2$$

where we have $T_0 > a\delta\hat{g}$.

To get into Linear Region of the H-D curve, we need Optical Density, D

$$D \approx 1 \quad \Rightarrow \quad T \approx 0.1$$

so 90% of **intensity** of reconstruction beam absorbed in the hologram.

Estimate of Efficiency: Transmitted light split between ± 1 order and DC terms.

$$\text{Reconstruction} \propto \left| \frac{ar^2}{g_0} o(x,y) \right|^2$$

$$\text{DC} \propto |T_0 r|^2$$

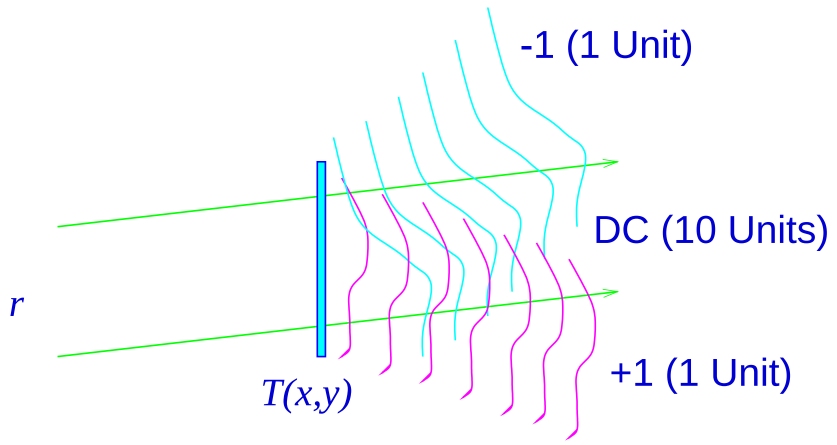
so substituting for T_O and a, we get ratio

$$\frac{\text{Reconstruction}}{\text{DC}} \approx \frac{\gamma^2}{4} \frac{|o|^2}{|r|^2}$$

so if $\gamma \approx 1.5$ and Object to Reference ratio ≈ 0.2 then

$$\frac{\text{Reconstruction}}{\text{DC}} \approx 0.1$$

We get an intensity split of



So only about $1/12$ of *transmitted* light goes into useful $+1$ order reconstruction.

So about $1/10$th of reconstruction beam transmitted by hologram, and about $1/10$th of that into useful reconstruction, so:

So total efficiency $\approx 1\%$

Possible to get $2 \rightarrow 3\%$ by use of *Toe* of H-D curve, also get some "thick hologram" effects that than improve things, but very difficult to exceed 5%.

2) Thin Phase Holograms

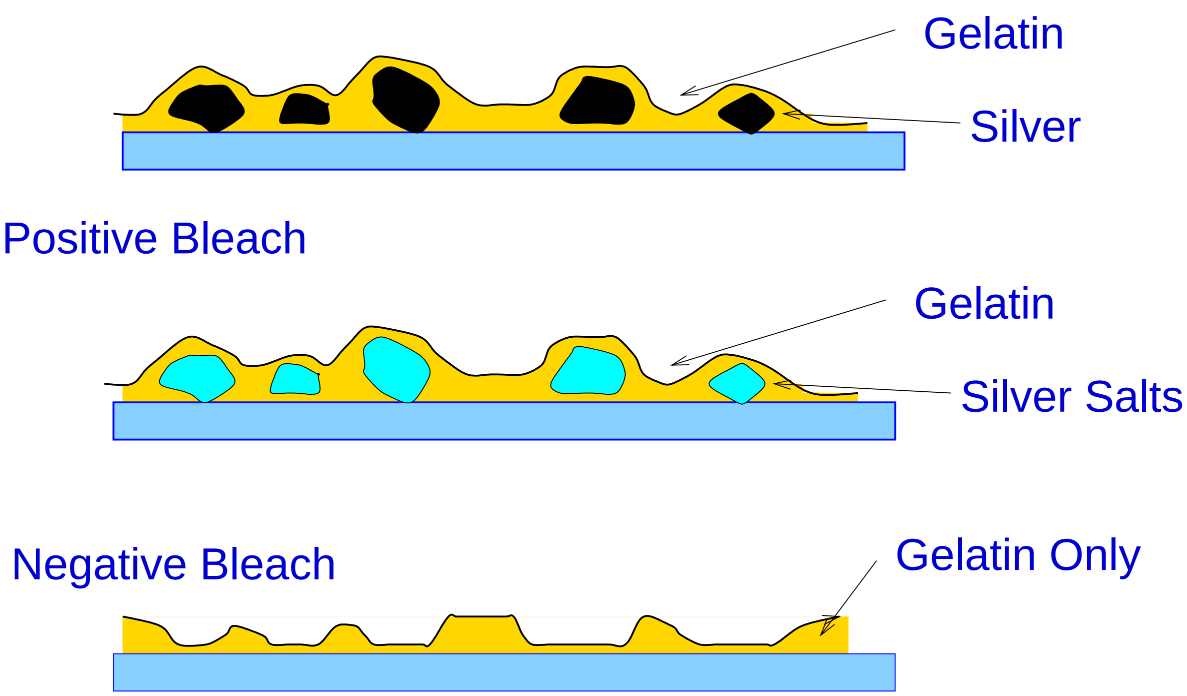
Modify the process so that the Amplitude transmittance is

$$T_a = \exp(\imath \Psi(x,y))$$

where $\Psi(x,y)$ is a monotonic function of $g(x,y)$ (typically non-linear). (See tutorial problem).

No Light Absorbed $\Rightarrow$ Brighter Reconstruction

Usual method is to bleach hologram,



Two bleach types:

1. **Positive Bleach** Replace silver with a transparent salt, phase different encoded in thickness of gelatin plus salts.
2. **Reversal Bleach** Remove silver and let gelatin fall to encode *negative* of phase distribution.

Process works, but again chemistry difficult, Problems are

- Stronger high order (non-linear) terms.
- Third (and Fourth) order terms become important.
- Noise (scatter) due to crystal structure of salts and cracking of the gelatin.
- Need to use strong reducing agents and/or hazardous organic solvents.

Expect about $\times 10$ efficiency due to no absorption, often able to be $20 \rightarrow 25\%$ with careful chemistry.

Maximum possible efficiency is 33%

Process used for holographic lenses and commercial systems, (see later).

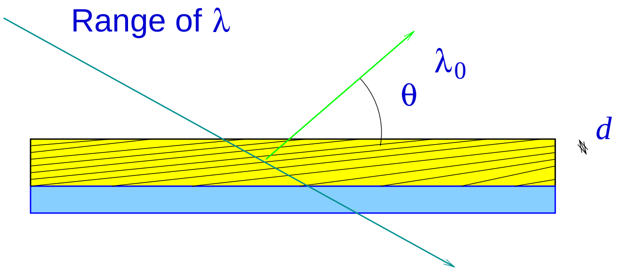
White Light Holograms

Thin hologram is like a diffraction grating.

$$\text{White Light} \Rightarrow \text{Spectrum}$$

Possible to use narrow band filter, but very inefficient.

Consider Three Dimensional Bragg Plane structure



We get strong reflection, if and only if,

$$\lambda_0 = 2d\sin\theta$$

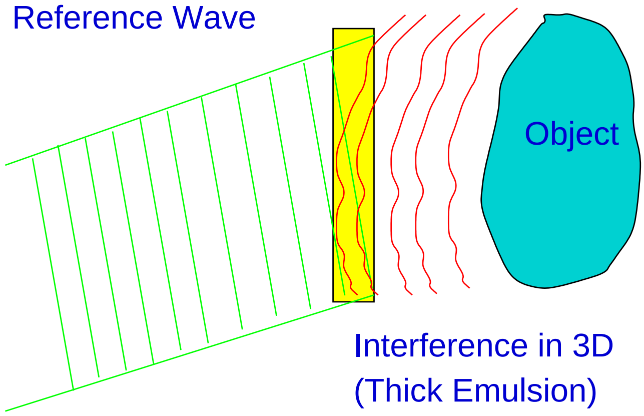
So Bragg Plane structure acts a a wavelength selector.

If we modulate shape of Bragg planes $\Rightarrow$ modulate amplitude/phase of reflected light. Hence we can make a hologram

Full mathematical theory possible, but beyond this course. (see references).

Formation of White Light Hologram

Form interference in Three Dimensions (standing waves in depth of emulsion)



Thick emulsion (15μm is typical).

$$\text{Bragg plane separation} \approx \lambda/2 \approx 300\mu\text{m}$$

Need very small silver grains, so special (very slow) holographic material.

Fringes much finer, so much greater stability problem.

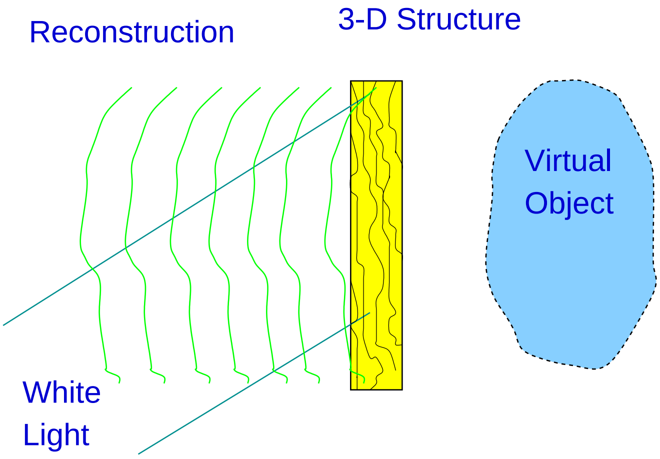
Need "thick" material, (typically 15μm, $\approx$ 40 Bragg planes being typical.)

Usually **Bleach** to get thick three dimensional (phase) structure, typically known as

Volume Hologram

Reconstruction of White Light Holograms

Illuminate which White Light.



Three Dimensional Bragg planes select single wavelength.

Modulation of planes gives reconstruction.

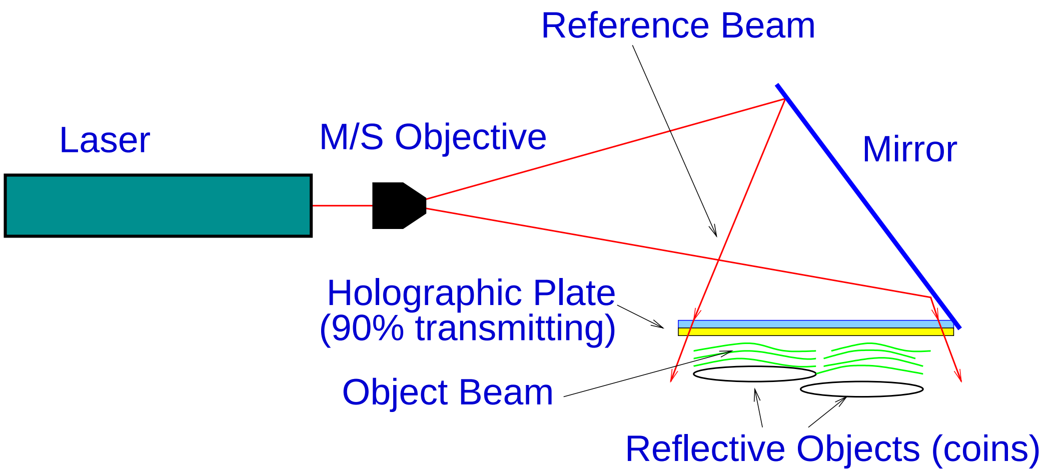
See Virtual Object behind plate

Able to get diffraction efficiencies up to about 80% at wavelength λ.

Note: Hologram is **Recorded** in Coherent (monochromatic) light, but can be **Reconstructed** in White (polychromatic) light.

Practical System

We can use fact that unexposed holographic plates are $\approx 90\%$ transmitting to give simple recording system,



The reference and object illumination beam are combined and the object beam is reflected back from the objects.

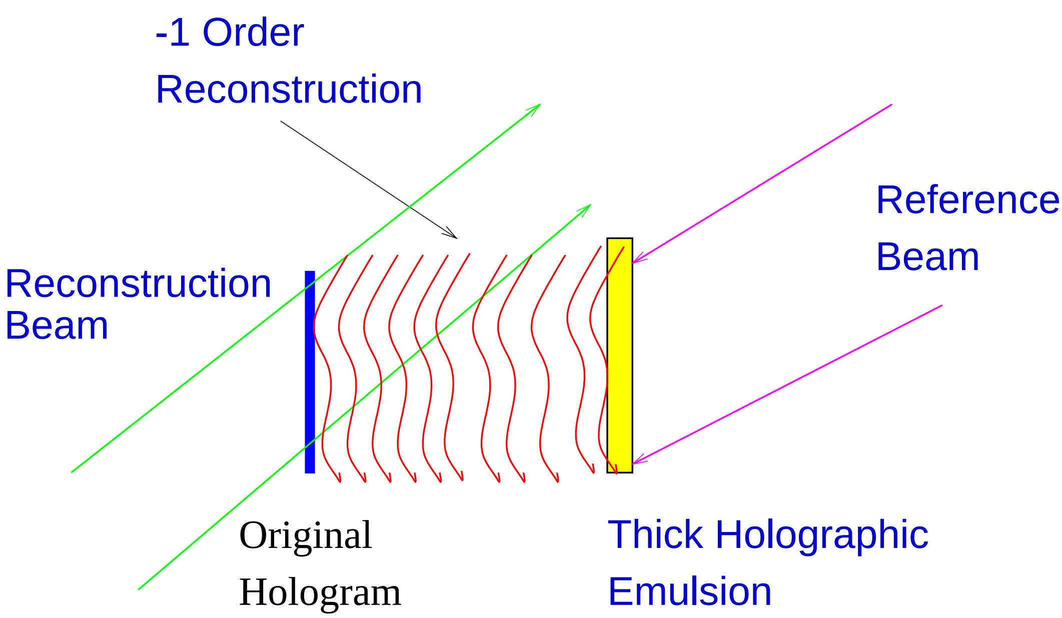
Simple system, but limited it depth of object since beam paths are not equal. For typical He-Ne laser works for objects up to about 3 cm in depth.

White Light Real Image

Like reconstruction to occur **in-front** of plate. No simple geometry (object get in the way).

Use a **Double Hologram** technique.

1. Make a simple off-axis thin hologram of object.
2. Use conjugate reconstruction as object beam for white light hologram.



On reconstruction the object "appears" in-front of the plate.

Mainly used for holographic display, eg, microscope hologram outside Room 4212

Silver Halide Material

Most common (in EC) by Agfa Geavert, (Holotest).

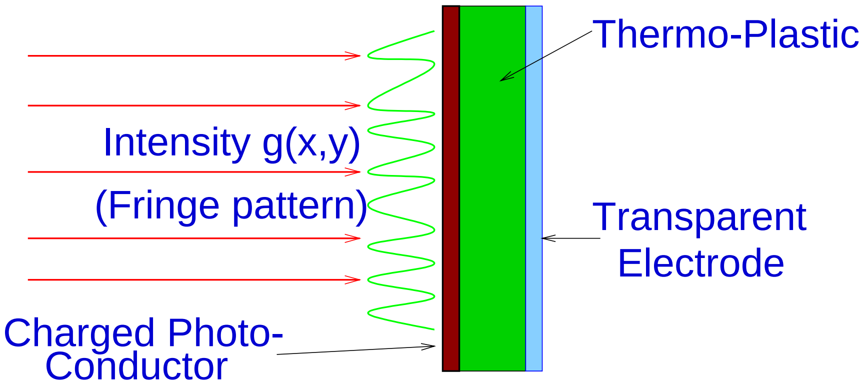
1. **10E75** Normal material for thin amplitude or phase holograms. ≈ 2000 lines/mm. Use with He-Ne.
2. **10E56** as 10E75 but for use with Argon lasers (514nm).
3. **8E75HD** Fine grain thick emulsion for white light holograms (high quality thin holograms). ≈ 6000 lines/mm. Use with He-Ne, ($\times 10$ less sensitive that 10E).
4. **8E56HD** as 8E75HD but for use with Argon lasers (514nm).

These are available as 35mm film, sheet film and plates up to 20×30".

Range of materials by Kodak (US only), and Russian suppliers.

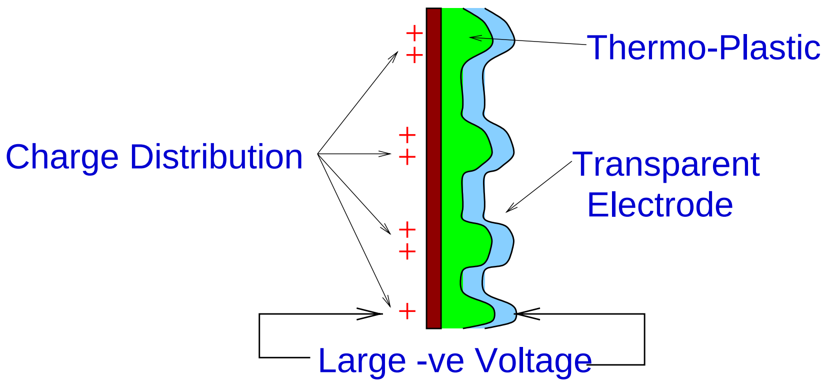
Best material developed in Russia due to major programme on white light display holograms to record art treasures.

Thermo-Plastic Plate:



Expose to intensity $g(x,y)$, surface change built-up proportional to the incident intensity.

Heat, (thermo-plastic become flexible)



Apply **Large** voltage, plastic distorts under electrostatic attraction.

Cool to "freeze" fringe pattern into the plate. (thin phase hologram)

1. **Re-usable** re-heat plate and discharge.
2. No wet chemicals, fast and easy.
3. Slightly less sensitive than silver halide.
4. Small plates $(30 \times 30\text{mm})$.

Dichromated Gelatin (DCG):
Emulsion of Potassium Dichromate and other chemicals in gelatin

Expose to light and "develop" (in IPA and Ethanol). Cross bonds formed in the actual gelatin that gives phase shifts.

- Very good for volume holograms (98% efficiency possible).
- Very insensitive (big Argon lasers)
- Chemical process not understood (black-art).
- Ultra sensitive to humidity

Used in expensive holographic systems.

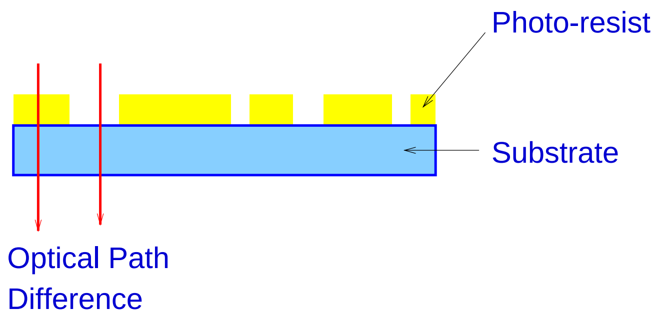
Photo-Polymers:
Synthetic DCG with cross links in polymer chains replacing cross-links in natural gelatin.

Two manufactures:

1. **Polaroid:** Similar to DCG, (80 $\rightarrow$ 90% efficiency). Still problem with humidity.
2. **Du-Pont:** Not as good DE, but is easy to handle and not sensitive to humidity. Rather insensitive to light.

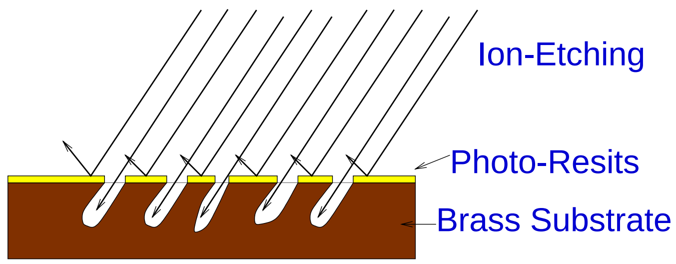
Photo-resist plus Embossing:
Photo-resist is a form of Perspex mainly used in semi-conductor industry. Sensitive to Blue and UV light.

Expose and develop to get (typically binary), thin phase hologram



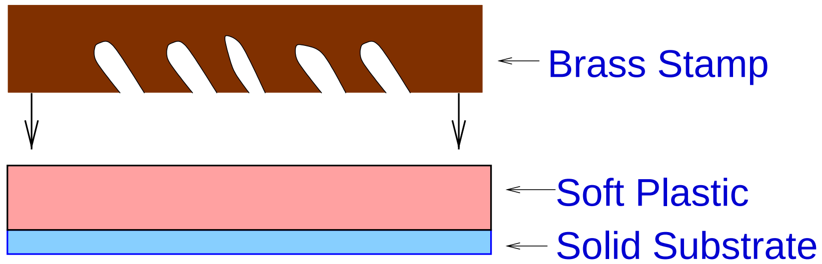
Either use as hologram (holographic lens), or more often.

Make Stamp:
Use a brass substrate, and then etch the pattern.

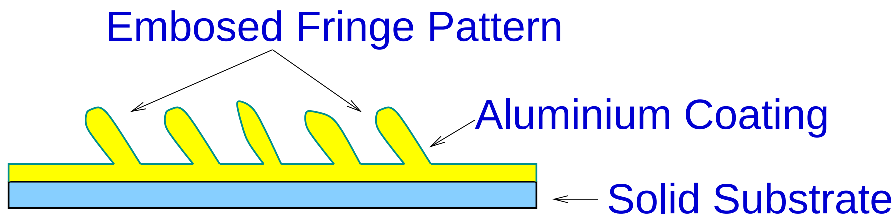


(If etch deep, get some 3-D Bragg effect, so partial white light possible).

Stamp holograms (exactly like CDs or records),



Coat with Aluminum to get reflective structure.



- Mass production process, so VERY cheap. (used on Credit Cards and advertising). 10p each.
- Security item, difficult (but NOT impossible) to copy.
- Poor quality, with limited 3-D effect. Not useful in optical systems, but major commercial use of holography.