7 Photographic Properties

Short set of questions on the photographic process all exploring the relation between exposure and transmittance. Questions 1, 2 and 4 are essential to the course.

7.1 Image of a disc

Show that for an image of a distant disc of constant intensity the intensity on the film plane is

$$\propto \frac{1}{\mathrm{F_{No}}^2}$$

and hence that the exposure

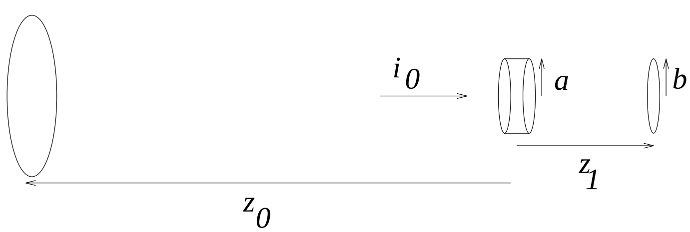
$$E \propto \frac{\tau}{\mathrm{F_{No}}^2}$$

where τ is the exposure time.

Hint: consider light from the object causing an approximately constant intensity across the aperture of the lens.

Solution

Consider the image of a distant disc,



If $z_0 \gg z_1$ then $z_1 \approx f$. If the object is distanct, then we get approximately constant intensity i_0 across the lens aperture, so the total energy passed by the lens is:

$$P = i_0 \pi a^2$$

If this forms a disc of radius b and intensity g in the image plane, then

$$P = g \pi b^2$$

so if there is no energy loss, then the intensity of the in the image is,

$$g = i_0 \frac{a^2}{b^2}$$

so we have that

$$g \propto a^2$$

so noting that

$$\mathrm{F_{No}} = \frac{f}{2a}$$

then we have that

$$g \propto \frac{1}{\mathrm{F_{No}}^2}$$

The exposure E on the film is the intensity $\times\tau$ the exposure time, so the exposure on the film is

$$E \propto \frac{\tau}{\mathrm{F_{No}}^2}$$

7.2 Film Gamma

For a film of $\gamma = 1.3$ the image of a region of constant intensity has an OD of 0.5 when photographed with an $\mathrm{F_{No}} = 4$ and an exposure time of $1/125$ of a second. What is the OD of the image of the object when photographed with an $\mathrm{F_{No}} = 16$ and an exposure time of $1/2$ second.

Hint: Use Q 7.1, whether you can show it or not.

Solution

The initial exposure is at

$$\mathrm{F_{No}} = 4 \quad \& \quad t_1 = \frac{1}{125}\mathrm{sec}$$

On the film we get an intensity g_1, so the expoure is

$$E_1 = g_1 t_1$$

The film gamma is $\gamma = 1.3$, and the optical density,

$$D_1 = \gamma \log_{10} E_1 - D_0 = 0.5$$

The second exposure is

$$\mathrm{F_{No}} = 16 \quad \& \quad t_1 = \frac{1}{2}\mathrm{sec}$$

since we have that

$$g \propto \frac{1}{\mathrm{F_{No}}^2} \quad \Rightarrow \quad g_2 = g_1 \frac{4^2}{16^2} = \frac{1}{16} g_1$$

So the second exposure is

$$E_2 = g_2 t_2 = \frac{125}{2} g_1 t_1 = 3.9 E_1$$

So the new optical density is

$$\begin{aligned} D_2 &= \gamma \log_{10}(3.9E_1) - D_0 \\ &= \gamma \log_{10} E_1 - D_0 + \gamma \log_{10} 3.9 \\ &= D_1 + \gamma \log_{10} 3.9 \\ D_2 &= 1.27\ \mathrm{OD} \end{aligned}$$

7.3 For Photographers

Your camera is equipped with a $F_{No} = 2$ lens, and you find that your normal speed film of 100ASA is giving you an exposure or 1/8th of a second with the aperture full open at $F_{No} = 2$. You decide that this film is too slow and change the film for a 1600ASA ultra fast film. If you set the camera speed to 1/60th of a second, what aperture setting should you use.

Hint: Use the fact that ASA is linear with Exposure and the result from question 7.1. Owners of a "real camera" (one where you can set film speed, aperture/exposure and check it with the built-in exposure meter), can can verify this result for themselves.

Solution

Deatils of this solution are beyond what is required for this course.

The exposure is given by $E = gt$, but we have from 7.1 that

$$E \propto \frac{\tau}{{F_{No}}^2}$$

The ASA number is *linear* with exposure, to get the same optical density on the on different types of film we must have that,

$$\text{ASA}\, E = \text{Constant}$$

so substituting for E we have that,

$$\frac{\text{ASA}\,\tau}{{F_{No}}^2} = \text{Constant} = E_{No}$$

which is known as the *Exposure Number.*

Aside: old cameras did not have automatic exposures but often had a separate light meter (photo-cell) that was calibrated in Exposure Number. This was then transferred to a circular slide-rule attached to the camera that give the correct speed/aperture combinations that would result in a correct exposure.

For 100 ASA film, with $t = 1/8$ sec and $F_{No} = 2$, we have $E_{No} = 3.125$. So for 1600 ASA file and $t = 1/60$ sec,

$${F_{No}}^2 = \frac{\text{ASA}\, t}{E_{No}} = 8.53$$

so that

$$F_{No} = 2.92 (\approx 2.8)$$

On "Real Cameras" the available F_{No} steps are given by

$$F_{No} = 1, 1.4, 2, 2.8, 4, 5.6, 8, 11, 16, 22, 32$$

which are steps of approximately $\sqrt{2}$ and the exposure times (or *shutter speed*) is in factors of (almost) 2, begin typically:

$$\frac{1}{4}, \frac{1}{8}, \frac{1}{15}, \frac{1}{30}, \frac{1}{60}\frac{1}{125}, \frac{1}{250}, \frac{1}{500}, \frac{1}{1000} \text{ sec}$$

Then for a given ASA film light level of given E_{No} then the same optical density will be obtained if the shutter speed in altered by one step up/down and the F_{No} is altered one step down/up. For example

$$\frac{1}{4}\text{sec}/F_{No} = 11 \Longleftrightarrow \frac{1}{60}\text{sec}/F_{No} = 2.8$$

7.4 Photograph of PSF

You want to form take a photograph of the ideal PSF of a lens so that the third subsidiary maxima is *just* above the fog level without saturating the central peak. The available film has a dynamic range (ΔD), of 2.2. Estimate the γ to which the film should be processed.

The third subsidiary maxima of the ideal PSF has an intensity of 0.16% relative to the central peak. (see section 3, OHP 12).

Solution

Take the central peak exposure to be E_0 and the third subsidiary ring to be E_3. We have that

$$\frac{E_3}{E_0} = 1.6 \times 10^{-3}$$

Assume that the exposure E_3 results in the "fog level" optical density, so that

$$D_f = \gamma \log_{10}(E_3) - D_0$$

while E_1 results in saturation, giving

$$D_s = D_f + \Delta D = \gamma \log_{10}(E_0) - D_0$$

so we have that

$$\Delta D = \gamma \log_{10}(E_0) - \gamma \log_{10}(E_3)$$

which gives γ to be

$$\gamma = \frac{\Delta D}{-\log_{10}\left(\frac{E_3}{E_0}\right)} = 0.79$$

This is a low γ (most films have $\gamma \approx 1.3$, so have to modify the γ during the development process by "long" exposure followed by "short/cold" development.

7.5 The Pinhole Camera (again)

You want to "actually" use the pinhole camera specified in question 7.1 to take a photograph using 400ASA photographic plate. On a bright sunny day you find that you obtain a good exposure with a "normal" camera using with an exposure time on $1/500$th of a second with the aperture set to $\mathrm{F_{No}} = 8$ when using a 100ASA film. What exposure time would you need with the pinhole camera under the same conditions and comment.

Solution

To get a good negative from the pinhole camera we need the same *Expoure Number* $\mathrm{E_{No}}$ as for the "real camera". We have solution 8.3 that,

$$\mathrm{E_{No}} = \frac{\mathrm{ASA}\ \tau}{\mathrm{F_{No}}^2}$$

so for the exposure values for the "real camera" we have that $\mathrm{E_{No}} = 3.125 \times 10^{-3}$.

From solution 7.1 a pinhole camera with a "focal length" of 10cm has as optimal pinhole diameter of approximately 0.25mm. It effective F_{No} is therefore

$$F_{No} \approx 400$$

so for 400 ASA film we get,

$$\tau = \frac{E_{No} {F_{No}}^2}{ASA} = 1.25\,\mathrm{secs}$$

which is a rather long exposure to be useful, in particular the pinhole camera would have to be mounted on a secure tripod and the scene would have to be stationary. However pictures with a simple pinhole camera *are* possible given a long enough exposure time.