5 Optical Measurements

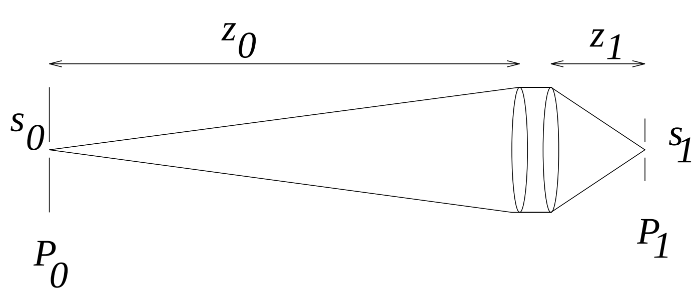
A short set of "practical" problems associated with the measurement of the properties of real optical systems. The computer simulation program is of particular interest and should be played with, again there is no actual programming required, you just run the program.

5.1 Line Scan Measurement

You wish to measure the OTF of a 80 mm, $F_{No} = 2.8$ camera lens by scanning the image of a slit with a 50:1 object to image distance. Sketch a suitable optical system and calculate suitable slit widths to use in the object and image plane.

Solution

Set the lens in a $50:1$ magnification system with a illuminated slit and a scanning slit infront of a detector.



Assume the, typically, multi-element camera lens can be approximated by a simple lens, so

$$z_0 = 50z - 1$$

so if $f = 80$mm, then from the lens formula, we get that

$$z_1 = 81.6\text{mm} \quad \& \quad z_0 = 4.08\text{m}$$

Lens is $F_{No} = 2.8$, so that lens diameter is $d = 28.6$mm.
Object Slit: Light rays are reversable, so the effective PSF in plane P_0 is of radius

$$r_0 = \frac{1.22\lambda z_0}{d} = 0.096\,\text{mm} \approx 100\,\mu\text{m}$$

if we assume $\lambda = 550$nm. Slit s_0 must be small than this resolution limit, typically half the size is suitable, so

$$s_0 \approx 50\,\mu\text{m}$$

This is about twice the thickness of a human hair and is fairly easy to make, usually with two knife edges. This is about the usable limit of simple varaible slits used in second year laboratory.

Image Slit: Here the size of the PSF in plane P_1 is,

$$r_1 = \frac{1.22\lambda z_1}{d} = 1.9 \times 10^{-3}\text{mm} \approx 2\mu\text{m}$$

so again we need s_1 to be less than this, so typically we want

$$s_1 \approx 1\mu\text{m}$$

Is *is* just about possible to make a 1μm slit by lithographic etching of chrome on glass as used in the semi-conductor industry, but it is very expensive, and not very practical.

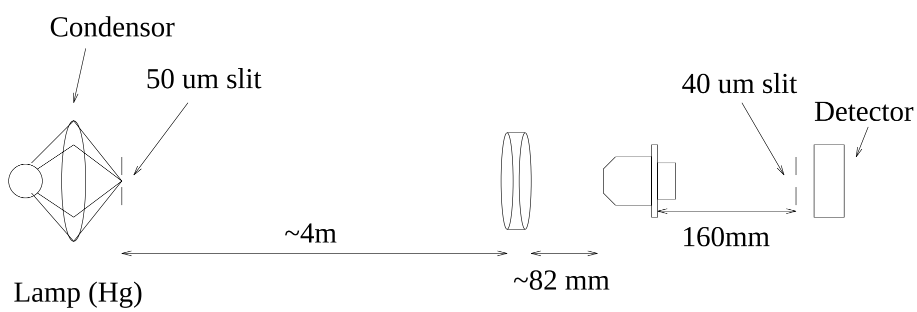
Better solution is to use a high quality microscope objective to "magnify" the effective slit. See question 1.4 for details of how to use a microscope objective. For example if we use a $\times 40$ objective, then if used with a 160 mm *tube length*, this will magnify the effective slit width by $\times 40$, allowing the use of a 40μm output slit.

Note: the microscope objective must have a much smaller PSF then the lens under test. From 1.4, we note that the *numerical aperture* of the lens under test is

$$\text{na} = \frac{1}{2\text{F}_{\text{No}}} = 0.18$$

while a *good* $\times 40$ objective will have a na $= 0.65$, so will have a much smaller PSF (as required).

So the final syste, will be



with the source slit illuminated by a incoherent source (for example a mercury lamp), and the 40μm slit translated infront of a large area photodetector to record the linescan.

5.2 Twyman-Green Interferometer

A high quality 300 mm focal length lens of diameter 50 mm is placed in a Twyman-Green interferometer with a spherical mirror and the system adjusted to give a single fringe. The spherical mirror is translated 0.05 mm along the optical axis. Calculate the wavefront aberration induced by this translation and sketch the expected fringe pattern.

Solution

Twyman-Green interferometer is adjusted to give one fringe, so system is initially "in-focus". Move lens ny 0.05 mm, but system is "double-pass" to the defocus distance

$$\Delta z = 0.1\text{ mm}$$

Defocus parameter,

$$D = \frac{1}{z-0} + \frac{1}{z_1} - \frac{1}{f}$$

but in a Twyman-Green (with collimated input), $z_0 \to \infty$ so that, provided that $\Delta z \ll f$, then

$$D \approx \frac{\Delta z}{f^2}$$

Noting that $\Delta W = 2Da^2$ where a is the lens radius, so that the *Wavefront Abberation* function,

$$W(r) = \frac{\Delta W}{2}\frac{r^2}{a^2} = \frac{\Delta z r^2}{f^2}$$

We also have that

$$\Delta W = \frac{2\Delta z a^2}{f^2} = 1.38 \times 10^{-3}\text{mm}$$

so if $\lambda = 500$nm then $\Delta W = 2.8\lambda$.
The phase shift is given by

$$\Phi(r) = \frac{2\pi}{\lambda} W(r) = \frac{2\pi \Delta z r^2}{\lambda f^2}$$

so the wave returned by the arm with the lens is $\exp(\imath\Phi(r))$, so after interference with the plane beam in the other are gives,

$$1 + \exp(\imath\Phi(r))$$

however we detect the intensity

$$|1 + \exp(\imath\Phi(r))|^2 = 2 + 2\cos(\Phi(r))$$

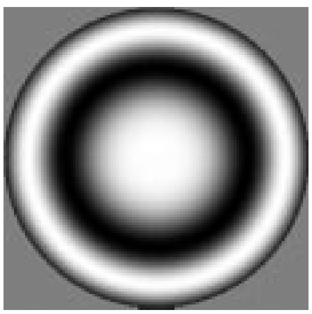
so we get a *Bright Fringe* when $\Phi(r) = \pm 2n\pi$, so we get fringes diameters of

$$r_n = \sqrt{\frac{n\lambda f^2}{\Delta z}}$$

so puting in the numbers we get

$$\begin{array}{lll} n = 0 & \to & r_0 = 0 \\ n = 1 & \to & r_1 = 21.2\text{mm} \\ n = 2 & \to & r_2 = 30\text{mm} \quad \text{Bigger than lens} \end{array}$$

so we get two bright fringes, one in the middle and one with a radius of 21.2 mm. Plot from *fringe* program is



5.3 Test Scheme

You a asked to devise a testing scheme for quailty control in manufacture of a low cost camera lens. The camera lenses have a focal length of 35mm and a $F_{No} = 3.5$. These leses are specified to have a OTF of better than 0.1 at 80 line/mm in the centre and 45 line/mm at the edge when used with a 35 mm film. The scheme should be able to test a lens in less than 1 minutes, and not use wet photographic process.

Hint: There is a large range of possible "correct answers"!.

Solution

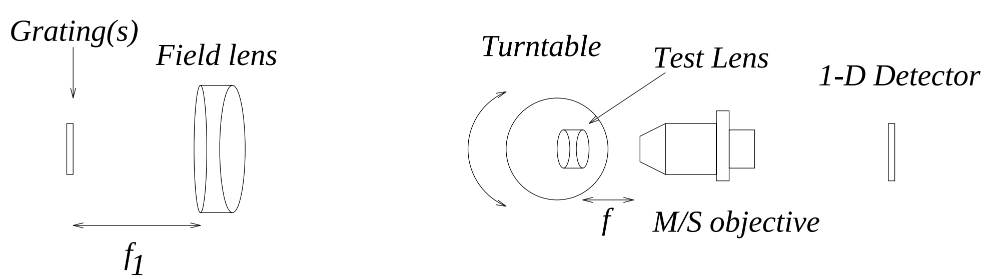
There are two simple methods of measuring the OTF of a lens, these being, 1): measure the PSF (or linescan), getting the OTF by taking a digital Fourier Transform, or 2): measure the OTF directly by imaging a range of gratings.

In this case you only need the OTF at 1 spatial frequency (in two positions), so the direct measure is quicker and easier to setup.

Points to note about the testing;

1. A camera lens is designed to operate with the object place at ∞, so it should be tested in this geometry. This is best done with a *field lens*. This lens must have a *much* wider OTF than the lens under test so as not to influence the measurement.
2. The lens must be tested at best focus, care has to be taken to ensure that the lens is placed correctly.
3. A 35mm negative is 24×36 mm so the maximum off-axis angle is $\pm 27.2°$.
4. You only need the OTF in one direction, so if tested with a vertical grating, you need a one-dimensional horizontal detector.
5. At 80 lines/mm the image is rather small and would usually need to be magnified with a microscope objective. Again this must not effect the image to any extent, so a high quality objective in needed.

A typical system would be



The one-dimensional detector would typically be a photo-diode array with a typical detector spacing of $\approx 25\mu$m, which is about 40 per mm, so to get an accurate measure of a 80 lines/mm pattern it must be multiplied by *at least* $\times 4$, $\times 10$ would be better.

The *field lens* would typically be a 300 mm $F_{No} = 5$ high quality imaging lens, which, for small targets, would give diffraction limited imaging.

The biggest problem is focus. Most mass produced lenses have a tolerance in the focal length of about $\pm 2\%$, which means that the best focus must be search for by moving the microscope and detector system while scanning the detector array for the best contrast image. This is best done under computer control. This process would be repeated with the lens rotated on the turntable, again under computer control.

This type of test system would be fairly easy to put together, but would be extremely expensive, due to the need for high quality field lens, microscope objective and accurate motorised stages. A reasonable costing for such a system would be £10k.

5.4 Fringe Program

Try the program `fringe` avaliable on the CP laboratory machines that calculate and display digitally simulated fringe patterns. The programme is located at:

```
wjh/mo/examples/fringe
```

Run program, and you will be prompted for aberrations, the fringe pattern is displayed by `xv` which allows you to save the fringe pattern.

Tasks to try with this programme:

1. Try various amounts of defocus and check that you get the fringe pattern you expect. You may have to use tilts to make the shape visible.

2. Examine the shape of the wave front at the Strehl limit for Spherical Aberration with cancelling defocus.

3. Look at the shape of other aberrations.