Tutorial Solutions

3 Point Spread Functions

This set of questions look in detail at the point spread function of lenses and its implications. The details of problems 6 and 7 are beyond the scope of this course but the results are very important since they put produce real practical numbers. The computer simulation programs in problem 8 are strongly recommended as they, hopefully, give insight into what a PSF looks like.

3.1 Bessel Functions

Given that the expansion of Bessel functions of integer order is,

$$\mathrm{J}_n(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (x/2)^{n+2k}}{k!\,\Gamma(n+k+1)}$$

write out the first **three** terms of the expansion for $\mathrm{J}_0(x)$ and $\mathrm{J}_1(x)$.
Use this expansion to determine the value of

$$\frac{\mathrm{J}_1(x)}{x} \quad \text{when } x = 0$$

Plot the functions $\mathrm{J}_0(x)$, $\mathrm{J}_1(x)$ and $\mathrm{J}_1(x)/x$ for $x = -10 \rightarrow 10$. (Maple would be a good idea).

Solution

Part a: Firstly note the identity, that for n an integer

$$\Gamma(n+1) = n!$$

so for we can write the expansion as:

$$\mathrm{J}_n(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (x/2)^{n+2k}}{k!(n+k)!}$$

So the first three terms are

$$\begin{aligned} \mathrm{J}_0(x) &= 1 - \frac{x^2}{4} + \frac{x^4}{64} \dots \\ \mathrm{J}_1(x) &= \frac{x}{2} - \frac{x^3}{16} + \frac{x^5}{192} \end{aligned}$$

So from these expansions, we get that

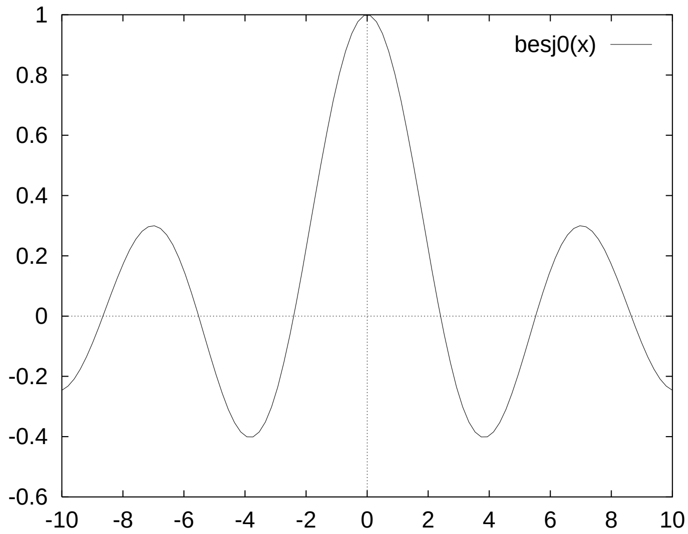
$$\frac{\mathrm{J}_1(x)}{x} = \frac{1}{2} - \frac{x^2}{16} + \frac{x^4}{192}$$

so that when $x = 0$,

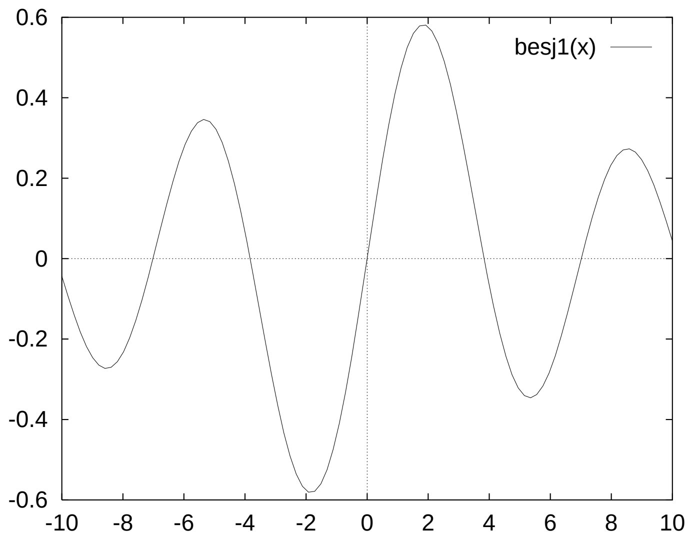
$$\frac{\mathrm{J}_1(0)}{0} = \frac{1}{2}$$

Part b: the plots are:

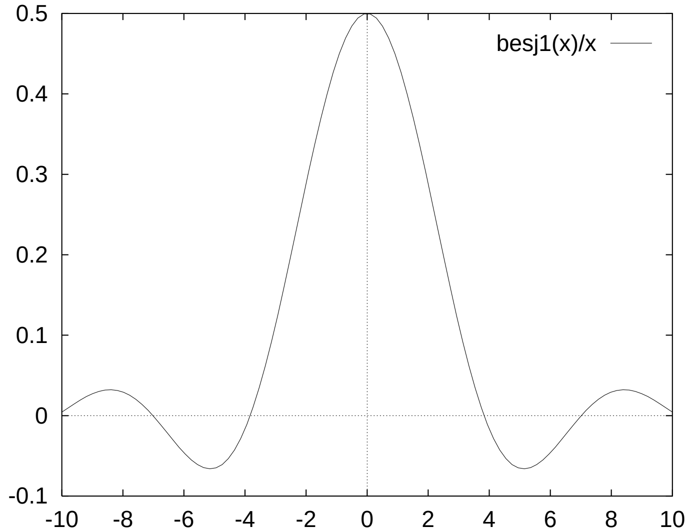
Plot of $\mathrm{J}_0(x)$ from $-10 \to 10$:



Plot of $\mathrm{J}_1(x)$ from $-10 \to 10$:



Plot of $\mathrm{J}_1(x)/x$ from $-10 \to 10$:



3.2 PSF of Simple Lens

Given that the Intensity PSF of a lens is given by the scaled Power Spectrum of the pupil function derive an expression for the PSF of a circular lens at geometric focus.

This is the full solution to the round lens outlined in lectures.

Solution

For a lens with pupil function $p(x,y)$ then if the object is a δ-function at ∞, the amplitude in the back focal plane is:

$$u_2(x,y) = \hat{B}_0 \int\int p(s,t) \exp\left(-\imath \frac{\kappa}{f}(xs + yt)\right) \mathrm{d}s\mathrm{d}t$$

where

$$\hat{B}_0 = B_0 \exp\left(\imath \frac{\kappa}{2f}(x^2 + y^2)\right)$$

the PSF is the intensity in the back focal plane, given by

$$g(x,y) = |u_2(x,y|^2$$

So for a simple round lens of radius a, we have

$$\begin{aligned} p(x,y) &= 1 \quad \text{for } x^2 + y^2 \leq a^2 \\ &= 0 \quad \text{else} \end{aligned}$$

If we shift to Polar Coordinates, with

$$s = \rho\cos\theta \quad \& \quad t = \rho\sin\theta$$

we get that

$$u_2(x,y) = \hat{B}_0 \int_0^a \int_0^{2\pi} \exp\left(-\imath \frac{\kappa}{f}(x\rho\cos\theta + y)\rho\sin\theta\right) \rho\,\mathrm{d}\rho\,\mathrm{d}\theta$$

We have that $p(s,t)$ is circularly symmetric, so $u_2(x,y)$ **must** also be circularly symmetric. *Note: if we rotate a circular lens we do not do anything to the PSF!* As a consequent, we can calculate $u_2(x,y)$ along *one radial* line. Select the line $y=0$, so we get,

$$u_2(x,0) = \hat{B}_0 \int_0^a \int_0^{2\pi} \exp\left(-\imath \frac{\kappa}{f} x \rho \cos\theta\right) \rho \, d\rho \, d\theta$$

We now use the standard result that

$$\int_0^{2\pi} \exp(\imath r \cos\theta) \, d\theta = 2\pi J_0(r)$$

so if we let

$$r = \frac{\kappa}{f} x \rho$$

we get that

$$u_2(x,0) = 2\pi \hat{B}_0 \int_0^a J_0\left(\frac{\kappa}{f} x \rho\right) \rho \, d\rho$$

now noting the second standard result that

$$r J_0(r) = \frac{d}{dr}(r J_1(r))$$

so by integration we have

$$r J_1(r) = \int_0^r J_0(t) t \, dt$$

If we substitute,

$$\alpha = \frac{\kappa}{f} x \rho$$

we can write

$$u_2(x,0) = 2\pi \hat{B}_0 \int_0^{\frac{\kappa x a}{f}} J_0(\alpha) \left(\frac{f}{\kappa x}\right)^2 \alpha \, d\alpha$$

This is now in the right form to be integrated, to get

$$u_2(x,0) = 2\pi \hat{B}_0 a^2 \frac{J_1\left(\frac{\kappa a x}{f}\right)}{\frac{\kappa a x}{f}}$$

The PSF is usually normalised so that $u_2(0,0) = 1$ and the phase term associated with $\hat{B}_0$ ignored, so that

$$u_2(x,0) = \frac{2 J_1\left(\frac{\kappa a x}{f}\right)}{\frac{\kappa a x}{f}}$$

Note the factor of 2, since $J_1(0)/0 = 1/2$ from previous question.

The system is rotational symmetry, so in two dimensions we have that

$$u_2(x,y) = \frac{2 J_1\left(\frac{\kappa a r}{f}\right)}{\frac{\kappa a r}{f}}$$

where $r^2 = x^2 + y^2$. The intensity PSF is then

$$g(x,y) = \left| \frac{2\mathrm{J}_1\left(\frac{\kappa a r}{f}\right)}{\frac{\kappa a r}{f}} \right|^2$$

which is the expressions given in lectures and in Physics 3 Optics course. (See next question for plots of this function).

3.3 Annular Aperture

Many telescopes, and mirror objectives have a central stop, giving an annular aperture. Extend the above derivation to give an expression for the PSF of annular lens.

Hint: The method is almost identical to the full aperture case except for the limits of integration.

For the special case of the central stop being half the diameter of the aperture find the location of the first zero, and compare this with full aperture case. Plot the cross-section of the intensity PSF of both apertures, and comment on the result.

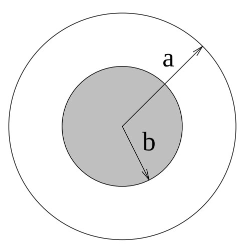
Hint: You will been a numerical solution to obtain the location of the first zero, and use Maple to produce the plots.

Solution

The annular aperture or outer radius a and inner b is described as

$$\begin{aligned} p(x,y) &= 1 \quad \text{for } b^2 \leq x^2 + y^2 \leq a^2 \\ &= 0 \quad \text{else} \end{aligned}$$

being of shape



The amplitude in the back focal plane is given by the *scaled* Fourier Transform of the pupil function. The main part of the integration is identical to the previous question for the open circular aperture except that the radial integration is between $b \to a$. The amplitude in tha back focal plane is now

$$u_2(x,0) = 2\pi\hat{B}_0 \int_b^a \mathrm{J}_0\left(\frac{\kappa}{f}x\rho\right)\rho\,\mathrm{d}\rho$$

This can separated and written as,

$$u_2(x,0) = 2\pi\hat{B}_0 \left[\int_0^a \mathrm{J}_0\left(\frac{\kappa}{f}x\rho\right)\rho\,\mathrm{d}\rho + \int_b^0 \mathrm{J}_0\left(\frac{\kappa}{f}x\rho\right)\rho\,\mathrm{d}\rho\right]$$

We now note that $J_1(x)$ is *even* so that $x J_1(x)$ is *odd*. Using this we can rearrange the integral to give

$$u_2(x,0) = 2\pi\hat{B}_0 \left[\int_0^a J_0\left(\frac{\kappa}{f} x\rho\right) \rho\, d\rho - \int_0^b J_0\left(\frac{\kappa}{f} x\rho\right) \rho\, d\rho \right]$$

Both terms here have the same form, that being identical to the form for the normal circular aperture. This can then be integrated to give,

$$u_2(x,0) = 2\pi\hat{B}_0 \left[a^2 \frac{J_1\left(\frac{\kappa a x}{f}\right)}{\frac{\kappa a x}{f}} - b^2 \frac{J_1\left(\frac{\kappa b x}{f}\right)}{\frac{\kappa b x}{f}} \right]$$

so again, we "spin" about the optical axis, normalise to get $u_2(0,0) = 1$ and ignore the phase term associated with $\hat{B}_0$ to get

$$u_2(x,y) = \frac{2}{(a^2 - b^2)} \left[a^2 \frac{J_1\left(\frac{\kappa a r}{f}\right)}{\frac{\kappa a r}{f}} - b^2 \frac{J_1\left(\frac{\kappa b r}{f}\right)}{\frac{\kappa b r}{f}} \right]$$

and then the intensity PSF is given by

$$g(x,y) = |u_2(x,y)|^2$$

Part b:
Look at locations of first zeros. For the full apetrure $b = 0$, we get the usual case

$$g(x,y) = \left| \frac{J_1\left(\frac{\kappa a r}{f}\right)}{\frac{\kappa a r}{f}} \right|^2 = \left| \frac{J_1(\alpha)}{\alpha} \right|^2$$

where $\alpha = \kappa a r / f$ The first zero occurs at $\alpha_0 = 1.22\pi$, so, as covered in lectures, the first zero is at

$$r_0 = \frac{0.61\lambda f}{a}$$

Look at the special case of $b = a/2$. By substitution we get that

$$g(x,y) = \frac{4}{(a^2 - a^2/4)^2} \left[a^2 \frac{J_1(\alpha)}{\alpha} - \frac{a^2}{4} \frac{J_1(\alpha/2)}{\alpha/2} \right]^2$$

which can we rewritten as:

$$g(x,y) = \frac{64}{9} \left[\frac{1}{\alpha} \left(J_1(\alpha) - \frac{1}{2} J_1(\alpha/2) \right) \right]^2$$

The first zero is thus when

$$J_1(\alpha) = \frac{1}{2} J_1(\alpha/2)$$

This has a numerical solution of

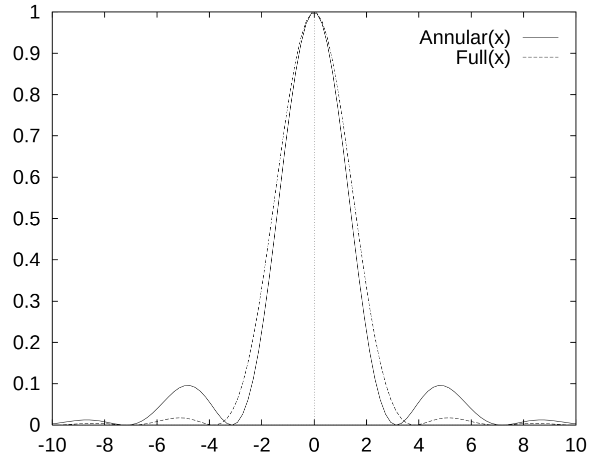
$$\alpha_0 \approx 3.144 < 1.22\pi$$

so the first zero occurs at

$$r_0 \approx \frac{0.5\lambda f}{a}$$

which is **narrower** then the full aperture case, so the PSF of the annular aperture in **narrower**, so the resolution is *increased*. This is **not** what you would expect!

The radial, normalsied, plot is



The full aperture is the wider plot with the lower secondard maximas, while the annular aperture has a narrower central peak but higher secondary maximas.

3.4 Resoltion of the Eye

Using the Rayleigh resolution criteria, estimate the angular resolution the the human eye. The quality of human vision is usually measured by a letter chart, and “perfect-vision” (usually referred to at 6:6 vision), is defined at the ability to read a letters who angular extent is 6’ of arc at a distance of 6 m. Is the human eye limited by diffraction?

Compare these results with the “number plate” criteria used in question 1.2.

Solution

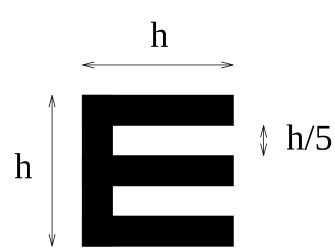
To a first approximation, the human eye can be consider as a simple lens than images onto a photo-sensitive surface, (the retina). In good light conditions, the diameter of the pupil is about 2.5 mm, and the peak sensitivity of the retina is 550 nm. The Rayleigh Resolution limit is given by

$$\Delta\theta = \frac{1.22\lambda}{d} = 2.7 \times 10^{-4}\text{Rad}.$$

converting to degrees we have that

$$\Delta\theta = 0.92' \approx 1'$$

A letter that subtends $6'$ of arc viewed from 6 m must be of height 10.5mm. The letters are usually “square”. In order to read the letter the observer must be able to resolve the components,



and in particular, able to resolve points that are about 1/5th of the height of the character. This estimate of the angular resolution of the eye is about $6/5 \approx 1.2'$ of arc. Both these estimates are very close.

Note: diffraction theory suggests that the larger the pupil the *better* the resolution. This is not true for the eye since in practice the resolution limit of the eye results from the separation cones on the retina, again at about $1'$ of arc. This resolution is obtainable under optimal conditions with high contrast letters that are well illuminated.

The "number plate at 25 yards" limit (see solution 1.2) gives letters of angular extend

$$\Delta\theta = 3.3 \times 10^{-3}\text{Rad} \approx 11'$$

which is about twice the resolution limit of the eye. This shows that the driving test eye criteria is not very strict. This however, is an easily obtainable resolution limit, and is a good working limit for cases such as question 1.2 where a person has be be able to "easily read" letters.

3.5 The Sparrow Limit

An alternative to the Rayleigh resolution limit is given by the Sparrow resolution limit which occurs when the "dip" between two adjacent PSF "just" disappears.

For an ideal PSF, find the separation when this limit occurs and discuss the practicallity of this limit.

Hint: Use MAPLE or GNUPLOT to plot ideal PSFs at varying separations until the "dip" just disappears.

Solution

We know from lectures or solution 2 that the ideal PSF is of given by,

$$g(x,y) = \left| \frac{2\mathrm{J}_1\left(\frac{\kappa a r}{f}\right)}{\frac{\kappa a r}{f}} \right|^2$$

which we can write as:

$$g(\alpha) = \left| \frac{2\mathrm{J}_1(\alpha)}{\alpha} \right|^2$$

where

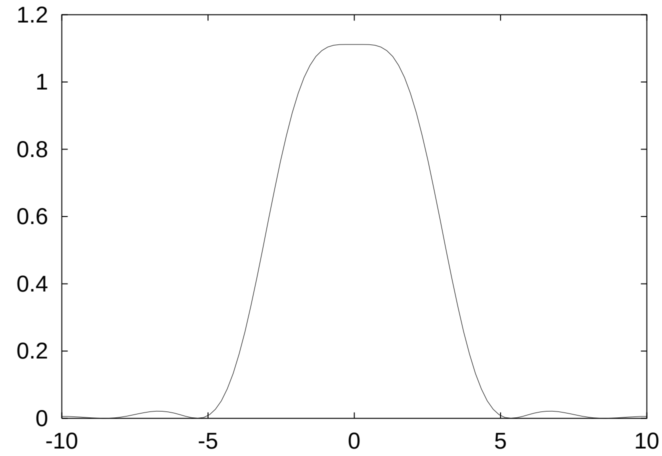
$$\frac{\alpha = \kappa a r}{f}$$

We know that the Rayleigh limit occurs at when the separation is given by the first zero of the Bessel function, as at $\alpha = 1.22\pi$ which gives a 27% "dip" between the peaks.

We can plot

$$g(\alpha - a/2) + g(\alpha + a/2)$$

for various a and see what happens, (you can do this yourself!), and you find that the "dip" just disappears with a plot as shown below



when $a \approx 2.99$, so giving a Sparrow separation of approximately

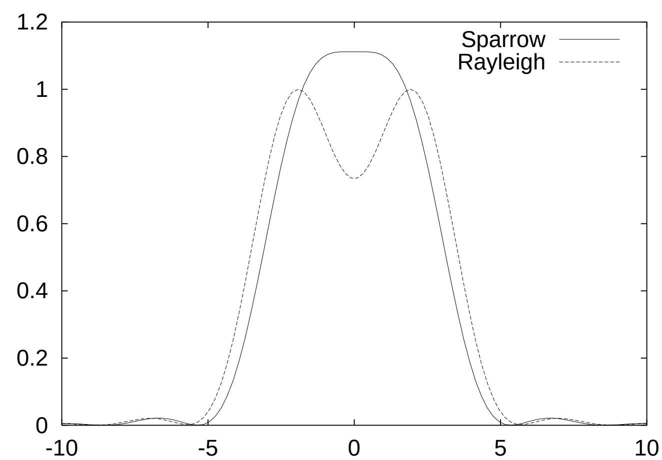
$$r_s = \frac{2.99}{2\pi}\frac{\lambda f}{a} = 0.952\frac{\lambda f}{d}$$

where $d = 2a$, the lens diameter. The Sparrow angular resolution is then given by:

$$\theta_s = 0.952\frac{\lambda}{d}$$

which appears to give better resolution than the Rayleigh limit.

If we compare the two limits plotted below,



we see that the Raleigh limit has a very distinct “dip” which is easy to measure while the Sparrow limit is narrower with a “flat top” to the intensity distribution which is much more difficult to measure. The Sparrow limit can be considered as an “absolute” limit while the Rayleigh limit is an easily obtainable limit. In particular we find that the Rayleigh limit is still obtainable by a system with aberrations to the Strehl limit while the slightest aberrations or noise in the detector system will result in two points at the Sparrow limit being interperated as a single unresolved point.

3.6 Location of Best Focus

Write down an expression for the wavefront aberration with in the presence of Spherical Aberration and Defocus. Show by minimisation that the optimal defocus occurs when

$$\Delta W = -\frac{7}{72} S_1$$

and hence the Strehl limit for Spherical Aberration, with defocus is

$$S_1 < 5.36\lambda$$

Solution

Without defocus the shape of the wavefront is

$$W(r) = \frac{1}{8} S_1 \frac{r^4}{a^4}$$

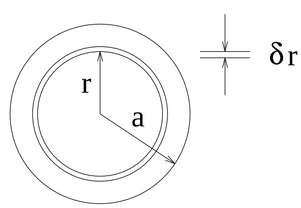
so the maximum phase error occurs at $r = a$, so Strehl Limit is,

$$\phi_{\mathrm{Max}} = S_1 \frac{\pi}{4\lambda} \leq \frac{\pi}{2}$$

so giving a Strehl Limit of $S_1 \leq 2\lambda$.
Now allow defocus, so the shape of the wavefront is now

$$W(r) = \frac{1}{8} S_1 \frac{r^4}{a^4} + \Delta W \frac{r^2}{a^2}$$

We can allow some the defocus to partially cancel out Spherical Aberration, as shown in lectures. To determine the best focus, we want to form a least squared error, but what do we want to minimise? The wavefront aberration function is rotationally symmetric, and "rings" of the aperture contribute in proportion to their area.



Ring of radius r, and thickness δr, the area

$$A = 2\pi r \delta r \quad \Rightarrow \quad A \propto r$$

to we need to weight the wavefront aberration function by r, so we need to minimise

$$\int_0^a |r W(r)|^2 \, \mathrm{d}r$$

To simplify the algebra, write

$$W(\hat{r}) = \alpha \hat{r}^4 + \beta \hat{r}^2$$

where $\alpha = \frac{1}{8}S_1$, $\beta = \Delta W$, and $\hat{r} = \frac{r}{a}$. So we get

$$\int_0^1 |\hat{r}W(\hat{r})|\,|^2 \mathrm{d}\hat{r} = \text{Minimum}$$

want to minimise with respect to β so that

$$\frac{\partial}{\partial\beta}\int_0^1 |\hat{r}W(\hat{r})|^2\,\mathrm{d}\hat{r} = 0$$

This gives us

$$\frac{\partial}{\partial\beta}\int_0^1 \left(\alpha\hat{r}^5 + \beta\hat{r}^3\right)^2 \mathrm{d}\hat{r} = 0$$

Expand the square, and differentiate, to give

$$\begin{aligned}\int_0^1 \frac{\partial}{\partial\beta}\left[\alpha^2\hat{r}^{10} + 2\alpha\beta\hat{r}^8 + \beta^2\hat{r}^6\right]\mathrm{d}\hat{r} &= 0\\ \int_0^1 \left[2\alpha\hat{r}^8 + 2\beta\hat{r}^6\right]\mathrm{d}\hat{r} &= 0\end{aligned}$$

We can now integrate to give

$$\frac{2}{9}\alpha + \frac{2}{7}\beta = 0$$

so that we have

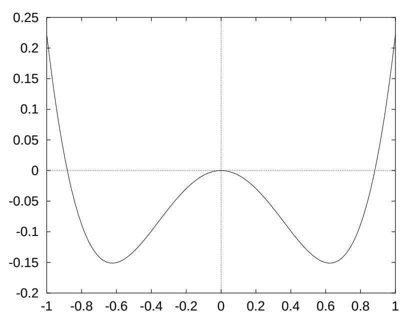
$$\beta = -\frac{7}{9}\alpha$$

now if we substitute back we get that

$$\Delta W = -\frac{7}{72}S_1$$

as quoted in lectures.

The shape of the wavefront is given by



so to define the Strehl Limit we need to find the minimum and maximum. These are located at

$$\frac{\mathrm{d}}{\mathrm{d}\hat{r}}W(\hat{r}) = 0$$

so at

$$\begin{gathered}\frac{\mathrm{d}}{\mathrm{d}r}\left(\alpha\hat{r}^4 + \beta\hat{r}^2\right) = 0\\ 4\alpha\hat{r}^3 + 2\beta\hat{r} = 0\end{gathered}$$

which either had solution $\hat{r} = 0$, or at

$$\hat{r}^2 = -\frac{\beta}{2\alpha}$$

but we have that $\beta = -\frac{7}{9}\alpha$, so we get the minimum at

$$\hat{r} = \sqrt{\frac{7}{18}} \quad \Rightarrow \quad r = \sqrt{\frac{7}{18}}a$$

From the graph, we see that this is the minimum, and the maximum is at $r = a$.
We know that the phase abberation is

$$\phi(\hat{r}) = \kappa W(\hat{r}) = \kappa(\alpha\hat{r}^4 + \beta\hat{r}^2) = \kappa\alpha\hat{r}^2\left(\hat{r}^2 - \frac{7}{9}\right)$$

So at $\hat{r} = 1$, we have at

$$\phi(1) = \frac{2}{9}\kappa\alpha$$

and at $\hat{r} = \sqrt{7/18}$ we have that

$$\phi\left(\sqrt{\frac{7}{18}}\right) = -\left(\frac{7}{18}\right)^2\kappa\alpha$$

so the maximum phase shift is

$$\phi_{\mathrm{max}} = \phi(1) - \phi\left(\sqrt{\frac{7}{18}}\right) = \frac{121}{324}\kappa\alpha \leq \frac{\pi}{2}$$

so we get that

$$\alpha \leq \frac{324}{484}\lambda$$

or in terms of S_1, we have that

$$S_1 \leq 5.36\lambda \quad \& \quad \Delta W = -0.52\lambda$$

which is much larger than the Strehl Limit.

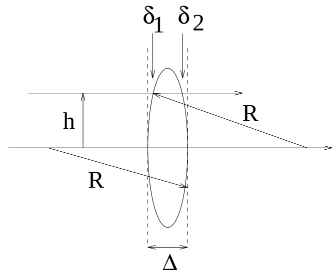
3.7 Spherical Aberration of a Simple Lens

A simple bi-convex lens has two spherical surfaces of the same curvature, refractive index $n = 1.51$ and focal length $f = 100$mm. Derive an expression for the Spherical Aberration introduced by this lens when used with an infinite object plane. Calculate the maximum diameter of the lens such that on-axis, the Spherical Aberration is within the Strehl limit.

Hint: calculate the phase function of the lens by polynomial expansion.

Solution

Take a simple biconvex lens of the same radius of curvature on both surfaces, so giving:



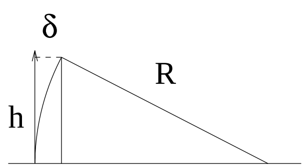
Phase delay of this lens is given by

$$\phi = \kappa n\Delta - \kappa(n-a)(\delta_1 + \delta_2)$$

but if the lens is "thin" and since $R_1 = R_2 = R$, then $\delta_1 = \delta_2 = \delta$, so ignoring the overall phase shift, we get

$$\phi = -2\kappa(n-1)\delta$$

Look at one spherical surface,



We have that

$$(R-\delta)^2 + h^2 = R^2$$
$$R^2 - 2\delta R + \delta^2 + h^2 - R^2 = 0$$
$$2\delta R - \delta^2 = h^2$$

If we take the traditional parabolic approximation, (ignoring δ^2), then be get

$$\delta_0 = \frac{h^2}{2R}$$

as we used in lectures. But to get information about the Spherical Aberration, we need to take a better approximation than this and retain the terms to *fourth* order.

Define a phase error ε so that

$$\delta = \delta_0 - \varepsilon$$

so that

$$2(\delta_0 - \varepsilon)R - (\delta_0 - \varepsilon)^2 = h^2$$
$$2\delta_0 R - 2\varepsilon R - \delta_0^2 + 2\varepsilon\delta_0 = h^2$$

Now noting that $2\delta_0 R = h^2$ and assuming that ε is small, so we can ignore ε^2, we get that

$$\delta_0^2 = 2\varepsilon(\delta_0 - R)$$

Now again assuming that $\delta_0 \ll R$, then

$$\varepsilon \approx -\frac{\delta_0^2}{2R} = -\frac{h^4}{8R^3}$$

so to *fourth* order the phase term of the surface becomes

$$\delta = \frac{h^2}{2R} - \frac{h^4}{8R^3}$$

so the total phase term of the lens becomes

$$\phi = -2\kappa(n-1)\left[\frac{h^2}{2R} + \frac{h^4}{8R^3}\right]$$

The first term is just the usual parabolic approximation, while the second term is associated with the Spherical Aberration. So the Spherical Aberration phase error is:

$$\Delta\phi = -2\kappa(n-1)\frac{h^4}{8R^3}$$

The Wavefront Abberation function is of the form,

$$\exp(\imath\kappa W(r))$$

so, noting that $r = h$ we have the

$$W(r) = -\frac{r^4}{4R^3}(n-1)$$

so again noting that the expression for Spherical Aberration is

$$\begin{aligned} W(r) &= \tfrac{1}{8} S_1 \tfrac{r^4}{a^4} \\ S_1 &= -\frac{2a^4}{R^3}(n-1) \end{aligned}$$

In term of focal length, we have that, if $R_1 = R_2$, then

$$\begin{aligned} \frac{1}{f} &= 2(n-1)\frac{1}{R} \\ R = 2(n-1)f & \end{aligned}$$

so we can substitute for R to get that

$$S_1 = -\frac{a^4}{4f^3(n-1)^2}$$

as a final expression.

Note that $S_1 \propto a^4$, to becomes a major problem as a become large.

Example: Strehl limit, with defocus, is $S_1 = \pm 5.36\lambda$ so if

$$f = 100\text{mm} \quad n = 1.47 \quad \lambda = 550\text{nm}$$

then the

$$\text{Radius } a = 7.14 \text{ mm} \quad \text{Diameter } d = 14.3 \text{ mm}$$

so for a focal length of 100 mm, a lens with $F_{No} > 7$ is within the Strehl Limit for Spherical Aberration.

In most practical optical systems with $F_{No} > 8$ Spherical Aberration is not a problem.

3.8 Computer Examples

Experiment with the program `psf` available on the CP laboratory machines to calculate the PSF of a round lens under various aberrations. The programme is located in:

`~wjh/mo/examples/pfs`

When you run the program you will be prompted for the various aberrations and then the PSF will be displayed via `xv`. You will then be given the option to save the horizontal linescan through the centre of the PSF in a format that can be used with `xgraph` or `xmgr`. There is **no** programming involved here, you just have to run the program!

Tasks to try with this programme:

1. Show that the psf of a annular aperture is slightly narrower than the corresponding open aperture.
2. Show that at the Strehl limit of defocus that the psf does not vary significantly in shape to the ideal psf. (Note at the Strehl limit $S_4 = 0.5$.)
3. Show that at with a defocus greater than the Strehl limit the psf become significantly wider.
4. Experiment with the off-axis aberrations an observe what happens to the shape of the psf.

Technical Details: The line scan in the intensity $h(x,0)$ but the image displayed by `xv` is actually

$$\log(h(x,y)+1)$$

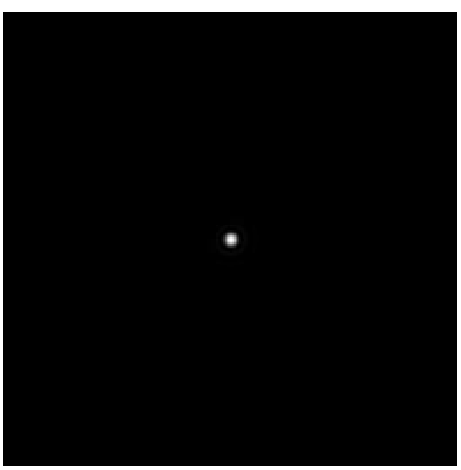
in order to reduce the dynamic range and make the outer ring structure more visible. However when viewing the ideal and Strehl images you will have to use the "Colo(u)r Edit" facility in `xv` to make the outer rings visible.

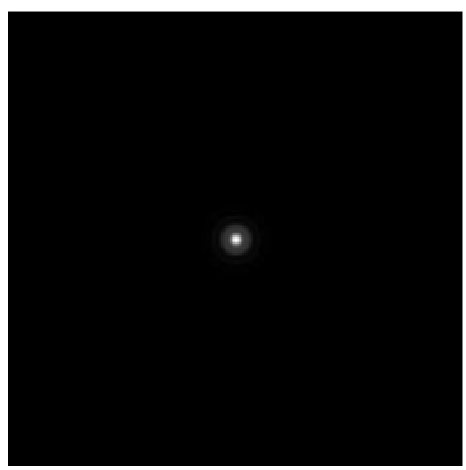
Solution

The results below show the PSF of a circular aperture under defocus. The decocus is specified in terms of S_4 which is

$$S_4 = 2\,\Delta W$$

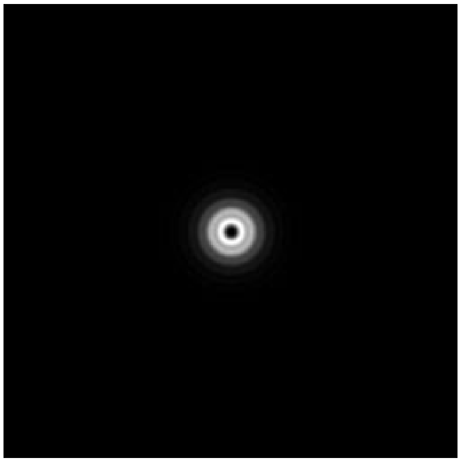
so the Strehl Limit for defocus is $S_4 = \lambda/2$.



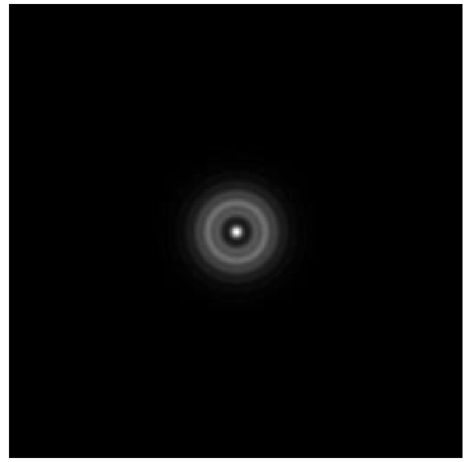


PSF with no defocus

PSF with $S_4 = \lambda$

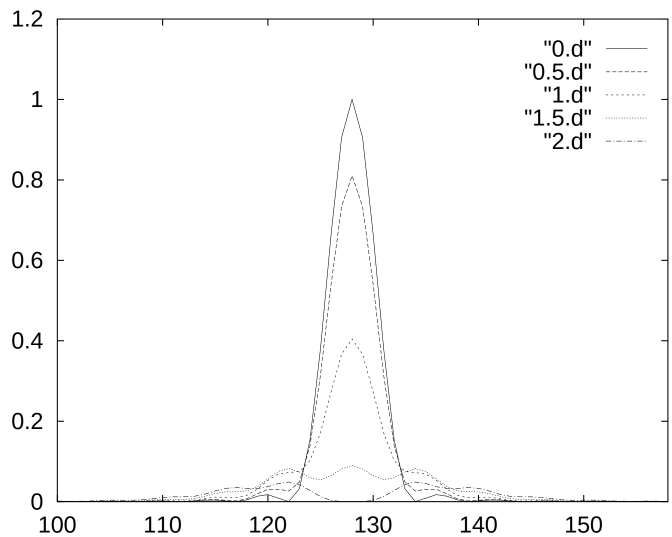


PSF with $S_4 = 2\lambda$.



PSF with $S_4 = 3\lambda$.

This can also be shown as a cross-section plot for $S_4 = 0, 0.5\lambda, \lambda, 1.5\lambda, 2\lambda$.



These plots show that the PSF is sharply peaked up to a defocus of $S_4 \approx \lambda$, (twice the Strehl limit), but then broadens very rapidly with further defocus. This suggests that the Strehl limit is a very strict limit and it is not until about twice the Strehl limit that defocus seriously degrades the PSF. Compare these results with the plots for the OTF in the next section.